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ABSTRACT

The role of negative instances in the acquisition of the mathematical concepts of commutativity and associativity of a binary operation was examined. Two levels of instruction (positive instances, and positive and negative instances) for commutativity and for associativity were crossed to form a 2 x 2 factorial design with 16 ninth grade subjects per cell. Criterion variables were number of correct responses, stimulus interval, and postfeedback interval during treatments and/or posttests. The results appeared to favor the treatments containing both positive and negative instances and supported the hypotheses that the effect of negative instances transferred from one concept to another. (Author/DT)

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Final Report

Project No. 1-E-021
Grant No. OEG-5-71-0025(509)

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NEGATIVE INSTANCES AND THE ACQUISITION OF THE MATHEMATICAL CONCEPTS OF COMMUTATIVITY AND ASSOCIATIVITY

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June, 1972

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ABSTRACT

The role of negative instances in the acquisition of the mathematical concepts of commutativity and associativity of a binary operation was examined. Two levels of instruction for commutativity (positive instances and positive and negative instances) and the same levels of instruction for associativity were crossed to form a 2 x 2 factorial design with 16 ninth grade subjects per cell. Criterion variables were number of correct responses, stimulus interval, and postfeedback interval during treatments and/or posttests. The results appeared to favor the treatments containing both positive and negative instances and supported the hypotheses that the effect of negative instances transferred from one concept to another. There were alternate explanations proposed for the results and further studies were urged.

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Richard J. Shumway

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PREFACE

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INTRODUCTION

Negative instances have been considered by mathematicians to be essential to the understanding of advanced mathematical concepts (Gelbaum and Olmsted, 1964; Steen and Seebach, 1970). Dienes (1964) argues for the use of negative instances in the teaching of mathematics to elementary and secondary school children. Bereiter and Engleman (1966) and Markle and Tiemann (1970) state explicitly that all instructional sequences designed for concept learning should include negative instances. Yet, Clark (1971), in a review of over 250 experimental studies in concept attainment, found 26 studies which reported a debilitating effect of negative instances on concept attainment and only 11 which did not support such a position. Markle and Tiemann (1972) state that, despite such laboratory evidence, positive and negative instances "are of equal importance in the teaching of real concepts." Bourne and Dominowski (1972) in a review of research support the contention that for simple, conjunctive concepts, a series of positive instances is to be favored over any mixture of positive and negative instances, but that for more difficult concepts, such as disjunction the advantage of positive instances is either absent or negative instances are favored. Both Clark (1971) and Bourne (1967), agree that the research in concept attainment is restricted in nature and that the concepts studied in schools are quite different. There is some evidence from recent educational research that negative instances are valuable components of classroom instruction (Tennyson, Woolley, and Merrill, 1971; Shumway, 1971).

The study is designed to investigate two questions about the role of negative instances in the acquisition of the mathematical concepts of commutativity and associativity of a binary operation.

1. What are the different effects of an instructional sequence of positive and negative instances and a sequence of all positive instances in the acquisition of commutativity and/or associativity?

2. Assuming there are effects for negative instances, do the effects of negative instances for one conept transfer to another concept?

Background

Mathematicians (Gelbaum and Olmsted, 1964; Steen and Seebach, 1970), mathematics educators (Dienes, 1964; Shumway, 1970), and educational psychologists (Bereiter and Engelman, 1966; Markle and Tiemann, 1970) have all advocated the use of negative instances for instruction.

Several studies involving concepts defined over finite universal classes using the dimensions of color, size, and shape have been carried out to investigate the role of positive and negative instances in the formation of simple concepts. In reporting one of the earliest of such studies, Smoke (1933) states that although there were no significant differences found in the rate at which concepts were learned when series of positive instances were compared with mixed series of both positive and negative instances, there was some evidence that negative instances tended to discourage 'snap judgments' on the part of the subjects during the learning of difficult concepts.

Hovland (1952) noted that the relative size of the class of positive instances and the class of negative instances introduced variability into the amount of information a particular instance communicated. Even when the amount of information instances communicated was equated, Hovland and Weiss (1953) found that more subjects completed their task successfully when the instances were positive than when the instances were a mixture of positive and negative instances or when the instances were all negative. Bruner, Goodnow, and Austin (1956) and others support the results of Hovland and Weiss (Glanzer, Huttenlocher, and Clark, 1963; Haygood and Devine, 1967; Mayzner, 1962).

Research has also shown that subjects seem to have both an inability and an unwillingness to use negative instances (Bruner, et al., 1956, Dominowski, 1968; Donaldson, 1959; Wojtaszek, 1971). It has been suggested that negative verbal information is simply more difficult for subjects to handle (Donaldson, 1959; Gough, 1965; Johnson-Laird and Tagart, 1969; Tavrow, 1966; Wales and Grieve, 1969; Wason, 1959). Others have suggested that the strategies used by the subjects do not seem to be compatible with effective use of negative instances (Bruner, et al., 1956; Braley, 1963; Conant and Trabasso, 1964; Davidson, 1969; Denny and Benjafield, 1969; Dervin and Deffenbacher, 1970; Duncan, 1964; Eifermann and Steinitz, 1971; Gough, 1965; Huttenlocher, 1967; O'Neill, 1969; Tagatz, et al., 1968; Wickelgren and Cohen, 1962).

In general, support of the theory that subjects are unfamiliar with the use of negative instances, Freibergs and Tulving (1961) have shown that although initial differences in median time to solution between subjects using positive instances and subjects using negative instances favor subjects using positive instances, these differences are virtually nonexistent at the end of a 20-problem series. The results of Chlebek and Dominowski (1970), Fryatt and Tulving (1963), Haygood and Stevenson (1967), Tavrow (1966), and Weber and Woodward (1966) appear to support the contention that subjects can learn to use negative instances as effectively as positive instances.

It has also been pointed out by Bruner et al. (1956) that the nature of the rule defining the concept is related to the role which negative instances play. Conant (1966) reports that for disjunctive concepts, mixed series of positive and negative instances are favored

over an all-positive series. Huttenlocher (1962, 1964) reports that for one-dimensional concepts, a mixed series of positive and negative series is favored over any other. Bourne and Guy (1968) report that in addition to confirming the results of Hovland and Weiss (1953) concerning the role of negative instances in conjunctive concept formation and the results of Hunt and Hovland (1960) concerning the relative difficulty of conjunctive, disjunctive, and conditional concept formation, it was also found that for rule learning (attributes given, rule unknown), subjects performed best on all rules when a mixture of positive and negative instances was presented. Recent research suggests that there is an interaction between usefulness of negative instances and the nature of the conceptual rule (Bourne and Ekstrand, 1969; Fraunfelker, 1971; Giambra, 1969; Krebs, 1970; Schaveneveldt, 1966; Shore and Sechrest, 1961; Taplin, 1971; Weber and Woodward, 1966).

In all of the investigations cited from experimental psychology the concepts studied were simple concepts defined over a finite universal class. It would seem appropriate to investigate concepts where the universal class was infinite and the concepts were of the more complicated type encountered in the study of mathematics. Wason (1960) reports that in using semi-mathematical concepts defined over an infinite universal class, the successful students were the ones who could suggest negative instances to test in the process of discovering a concept.

A second observation concerning the investigations cites is that the training sequence is somewhat different than that encountered in school learning. The subjects were given a series of instances and were asked to discover the concept by examining the instances. It is common procedure in school learning to first present the subject with the definition of the concept and then examine a series of instances in order to learn the concept (Ausubel, 1968). Frase (1972) supports this point by making a distinction between concept definition and concept formation.

Markle and Tiemann (1972) and Gagné (1965) suggest that in school learning the subject is to classify new instances which have not previously been seen. Clark (1971) describes the critical differences between the concept attainment tasks of experimental research and concepts in the classroom and calls for research in the content areas to bridge the gap. Cronbach (1957), Pereboom (1971), and Tagatz, Meinke, and Lemke (1968) have also called for studies in specific content areas. Some examples of such studies are those of Markle and Tiemann (1972), Marine (1972), and Tennyson, Woolley, and Merrill (1971) supporting the use of positive and negative instances and Dossey (1972) favoring positive instances over negative instances. In a study by this author, series of all positive instances and series of mixed positive and negative instances were used with mathematical concepts defined on infinite universal classes in the usual classroom setting where the instances were preceded by a definition of the concept (Shumway, 1970). The results suggest that when subjects are tested on a concept with new

instances, positive and negative instances are favored over subjects trained with positive instances alone. An alternate explanation could be that negative instances simply taught the subjects to be skeptical, independent of the particular concept being learned. It is the goal of the present study to determine whether negative instances have an effect on concept learning and whether this effect transfers from one concept to another.

Hypotheses

The first requisite is a definition for concept. As Kendler (1964) has pointed out, none of the many definitions of concept given in the literature can be considered totally adequate since they do not adequately characterize such concepts as color, time, space, etc. For the purposes of an experimental study, the most useful definitions are those given by Hovland (1952), Kendler (1961), Hunt (1962), Bourne (1966), and others in the field of experimental psychology. We choose to use components of the definitions of Kendler and Hunt.

A concept is a partitioning of a class X into two disjoint classes X_1 and X_2. The elements of the class X_1 are called positive instances of the concept and elements of the class X_2 are called negative instances of the concept. The class X is called the universal class over which the concept is defined. The notion of partition requires that the class X be the union of the class X_1 and the class X_2 and that the classes X_1 and X_2 are disjoint. To say that a subject knows the concept over the class X is to say that given any object from the class X the student is able to identify the object as a member of the class X_1 or the class X_2 associated with the concept over the class X. For example, a subject knows the concept pencil over the class of all objects in a particular schoolroom; if given any object from that schoolroom, the student is able to identify the object as either a pencil (an element of X_1) or a non-pencil (an element of X_2). The definition requires that this can be done with every object in the given schoolroom (every element of X). In analyzing the definition formally, it is clear that one need only be able to identify the elements of two of the three classes X, X_1, and X_2 in order to know the concept over the class X. This follows from the results that

$$X = X_1 \cup X_2,\ X_1 = X - X_2,\ \text{and}\ X_2 = X - X_1.$$

This observation would seem to suggest three possible strategies for the maximally efficient learning of a given concept over a class X with partition X_1, X_2. Namely, the student would learn to identify the classes

$$X \text{ and } X_2, \text{ or } X \text{ and } X_2, \text{ or } X \text{ and } X_1.$$

If one allows for redundancy, a fourth strategy can be added; the student would learn to identify the classes

$$X,\ X_1,\ \text{and}\ X_2.$$

Negative instances play a role in three of the four strategies.

The major problem investigated was the effect that negative instances have on the formation of mathematical concepts and if that effect transfers from one concept to another. Two mathematical concepts over the infinite class of all binary operations on the sets of natural, integral, and rational numbers were used. Concept A was commutativity of a binary operation and Concept B was associativity of a binary operation. All treatments for the acquisition of Concepts A and B were a series of instances which the subject was required to classify as a positive or negative instance of the concept. The treatment for Concept A consisting of only positive instances was denoted by A+, the treatment consisting of positive and negative instances was denoted by A±. The treatments B+ and B± were defined similarly. Figure 1 specifies the four treatment groups.

	A+	A±
B+	A+B+	A±B+
B±	A+B±	A±B±

Figure 1. Treatment groups

The class X for Concepts A and B is infinite. Each of the four treatments specified in the above table consisted of 40 instances, 20 for Concept A and 20 for Concept B. In the cases where both positive and negative instances were presented, 10 were positive and 10 were negative. The treatments were administered by IBM 2741 computer terminals.

The following definitions were made:

Binary operation: A binary operation * on a set S is a correspondence which associates with every ordered pair (a,b) of elements of S a unique element a * b of S.

Commutative: A binary operation * on a set S is said to be commutative if and only if for every a and b in S,

$$a * b = b * a.$$

Associative: A binary operation * on a set S is said to be associative if and only if for every a, b, and c in S,

$$a * (b * c) = (a * b) * c.$$

Stimulus interval: The stimulus interval is the length of time the stimulus is available to the subject for inspection.

Postfeedback interval: The postfeedback interval is the length of time between the presentation of the feedback and the occurrence of the next stimulus.

Delay of informative feedback: The delay of informative feedback is the length of time between the subject's response to a question and the presentation of the feedback associated with the subject's response.

A sample instance during the treatment follows:

Stimulus:

1. $a \circ b = 2 + b + a, \quad 3 \circ 4 = 9, \quad 0 \circ 1 = 3.$

$a \circ b = b \circ a$?

Response:

y

Feedback:

Correct.

Response:

Hit 'return key' to receive next stimulus.

The stimulus interval was taken to be the length of time between the end of the typing of the stimulus, i.e., the symbol "?," and the entering of the symbol"y," the response. There was no delay of the informative feedback. As soon as the response was entered, the

feedback was typed. The postfeedback interval was taken to be the length of time between the typing of the feedback and the subject's hitting of the return key to receive the next stimulus.

The following hypotheses were tested: There is no significant interaction or main effects for levels of A (A+, A±) and levels of B (B+, B±) in:

I. Mean number of correct identifications for

   a) criterion measure for Concept A;
   b) criterion measure for Concept B.

II. Mean total stimulus interval and postfeedback interval during

   a) treatment for Concept A;
   b) treatment for Concept B;
   c) criterion measure for Concept A;
   d) criterion measure for Concept B.

METHODS

Subjects, Design, and Treatments

A random sample of 85 of 410 ninth grade students from Clinton Junior High School were identified as the subject pool from which 64 subjects were randomly assigned in equal numbers to four treatments. Appendix A.6 documents the subjects dropped and the reasons for the drops. Appendix A.7 briefly describes the pilot studies and the time schedule of the experiment. The mean verbal I.Q. on the California Short Form Test of Mental Maturity for the students of Clinton was 105. Clinton students averaged approximately one grade level above the average for the total Columbus Public School population in standardized measures of reading, language, and arithmetic (Merriman, 1969).

Concept A was defined to be commutativity of a binary operation and Concept B was defined to be associativity of a binary operation. The symbol A+ denoted a treatment of 20 positive instances of Concept A and the symbol A± denoted a treatment of 10 positive instances and 10 negative instances of Concept A. The symbols B+ and B± were defined, similarly. Figure 2 specifies the 2 x 2 design matrix. Each treatment consisted of 40 instances in a fixed but random order. Each of the four treatments is given in full in Appendix A.4.

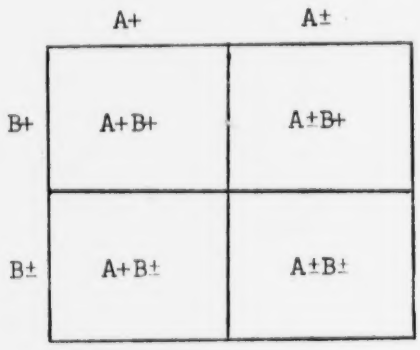


Figure 2. Design matrix

Table I specifies the number and type of instances for each treatment.

TABLE I

NUMBER AND TYPE OF TREATMENT INSTANCES PER CELL
AND NUMBER OF SUBJECTS PER CELL

Cell	Treatment	Number and Type of Instances				Total	Number of Subjects Per Cell
		Concept A		Concept B			
		Positive	Negative	Positive	Negative		
11	A+B+	20	0	20	0	40	16
12	A+B+	10	10	20[a]	0	40	16
21	A+B±	20	0	10	10	40	16
22	A±B±	10	10	10	10	40	16

[a]Item fourteen for cell 12 was scored as a positive instance and the subjects received feedback which identified the instance as positive. In fact, however, the instance was negative and some would argue that cell 12 received 19 positive instances of Concept B and 1 negative instance.

A sample instance from treatment A+B+ follows:

Stimulus:

1. $a \circ b = b + a + 2$, $3 \circ 1 = 6$, $4 \circ 5 = 11$.

$(a \circ b) \circ c \quad a \circ (b \circ c)$?

Response:

n

Feedback:

Incorrect.

Response:

'Return key'.

Instruments

The treatments were administered with an IBM 360/50 computer and IBM 2741 computer terminals. The programming language was Coursewriter III, version 2 (IBM, 1969). Stimulus intervals and postfeedback intervals for each item were recorded as well as the students' responses. Sample student interaction and computer programming for each treatment are displayed in Appendix A.4.

A random sample of 10 of the 20 binary operations used for the posttests (POA and POB) were used for the calculational pretests PCA and PCB. Stimulus intervals were recorded during both pretests.

The items for PCA and PCB were randomly ordered in pairs, operation by operation. A sample item of PCA and PCB follows:

Stimulus:

$a \circ b = 2 * (a - b)$, $3 \circ 1 = 4$, $7 \circ 2 = 10$,

3. $5 \circ 2 = ?$ (PCB)

Response:

6

Stimulus:

4. $9 \circ (7 \circ 3) = ?$ (PCB)

Response:

2

A complete list of items for the pretests PCA and PCB are given in Appendix A.3. Table II gives reliability estimates for the 64 subjects of the experiment. Both PCA and PCB proved to have reliability estimates in excess of .80.

TABLE II

RELIABILITY ESTIMATES FOR PRETESTS

Instrument	Number of Items	Reliability Estimate[a]
PCA + PCB	20	.89
Subtests:		
PCA	10	.81
PCB	10	.88

PCA + PCB - Pretest, Calculations without and with parenthesis
PCA - Subscore of Pretest, Calculations without parenthesis
PCB - Subscore of Pretest, Calculations with parenthesis
[a]Kuder-Richardson Formula 20 reliability estimate.

The posttests for Concept A and Concept B, POA and POB consisted of 20 binary operations not in any of the treatments. A sample item of POA and POB follows:

Stimulus:

1. $a \circ b = 2(a - b)$, $3 \circ 1 = 4$, $5 \circ 2 = 6$.

$(a \circ b) \circ c = a \circ (b \circ c)$?

Response:

y

Stimulus:

$a \circ b = b \circ a$?

Response:

y

Stimulus intervals were recorded for all items of POA and POB. A complete list of items for the posttest POA and POB may be found in Appendix A.4. Table III gives the number and type of instances for POA and POB, as well as the reliability estimates of .54 and .45. A possible explanation for the magnitude of these reliabilities will be given in the Conclusions section.

TABLE III

NUMBER OF INSTANCES, TYPE OF INSTANCES, AND RELIABILITY ESTIMATES OF POSTTESTS FOR CONCEPT A AND CONCEPT B (POA AND POB)

Instrument	Number of Instances			Reliability Estimate[a]
	Positive	Negative	Total	
POA	10	10	20	.544
POB	7[b]	13	20	.454

[a]Kuder-Richardson Formula 20 reliability estimate.
[b]Positive and negative instances are unbalanced because of the difficulty in finding simple, commutative but not associative binary operations.

Figure 3 gives a flow chart of the complete experimental sequence. There were three sessions at the computer terminal. The first, lasting approximately 30 minutes, consisted of an introduction to binary operations and pretests PCA and PCB. The second session, usually the following day, consisted of a brief introduction and one of the four experimental treatments and took approximately 25 minutes. The third session, usually on the third day, consisted of the two posttests, POA and POB and took approximately 25 minutes. In all cases students completed all three sessions within seven days. The four computer terminals were placed in an acoustically prepared room at Clinton Junior High School. Students reported to the computer terminal during their study hall or in some cases their mathematics class. One or two monitors were available at all times to insure proper operation of the computer terminal and to detect any deviations in terminal performance. Approximately 20 students were processed per week from January 17, 1972 to February 25, 1972. Students were randomly assigned to treatments before being scheduled for the administration of the experiment.

For the experiment, the independent variables were:

1. Levels of A (A+ or A±)
2. Levels of B (B+ or B±

3. PCA - Pretest calculations without parentheses;
4. PCB - Pretest calculations with parentheses;

5. PCSIA - Total Stimulus Interval for PCA;
6. PCSIB - Total Stimulus Interval for PCB;

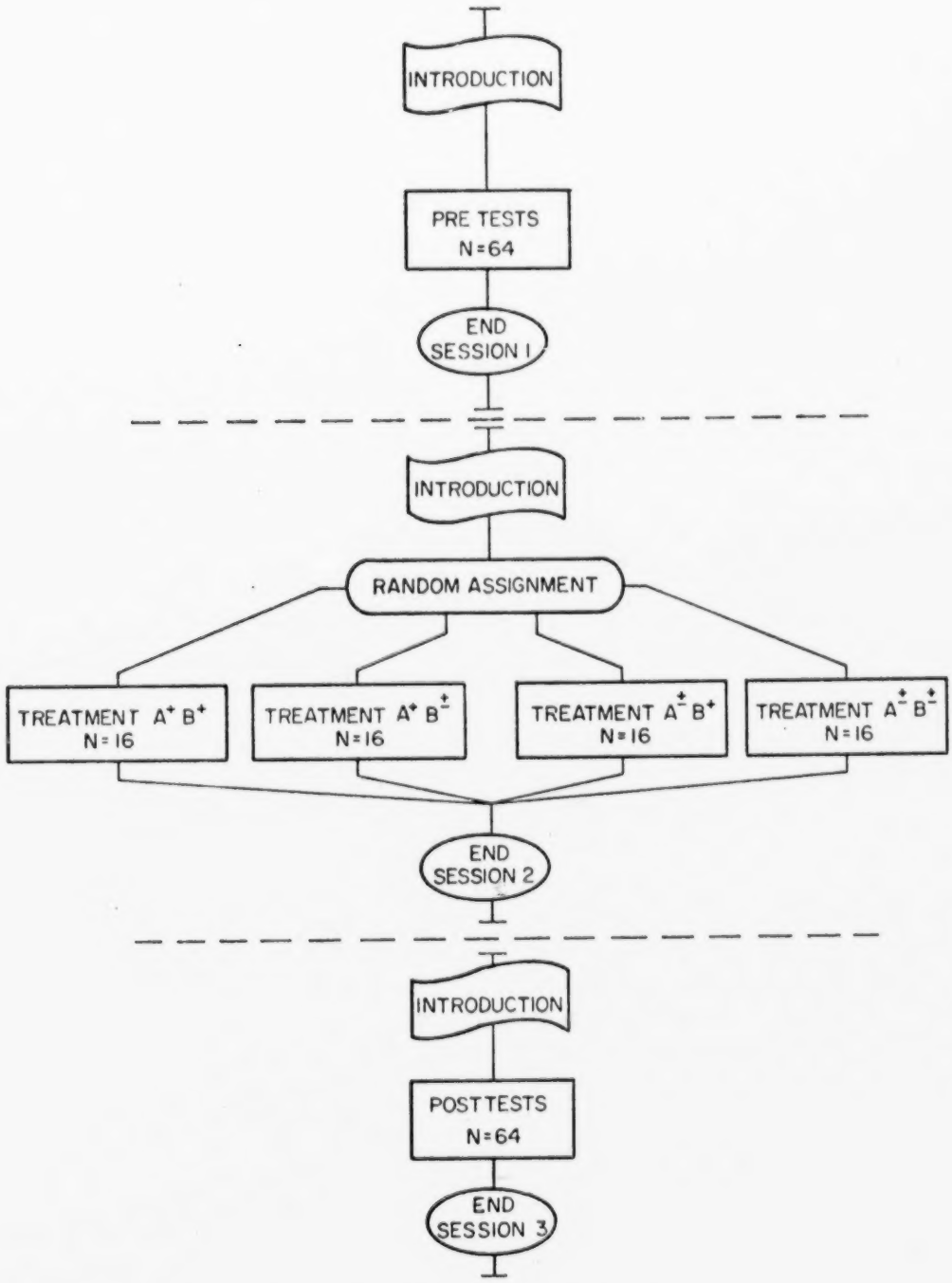


Figure 3. Flow chart of experiment

and the dependent variables were:

1. POA - Posttest for Concept A;
2. POB - Posttest for Concept B;

3. POSIA - Total Stimulus Interval for POA;
4. POSIB - Total Stimulus Interval for POB;

5. TSIA - Total Stimulus Interval during Treatment for Concept A;
6. TSIB - Total Stimulus Interval during Treatment for Concept B;

7. TPIA - Total Postfeedback Interval during Treatment for Concept A;
8. TPIB - Total Postfeedback Interval during Treatment for Concept B;

The design may be diagrammed as follows:

$$R_1 \quad O_1^A O_2^A O_1^B O_2^B \quad (X_1 \; O_3^A O_4^A O_3^B O_4^B) \quad O_5^A O_6^A O_5^B O_6^B$$

$$R_2 \quad O_1^A O_2^A O_1^B O_2^B \quad (X_2 \; O_3^A O_4^A O_3^B O_4^B) \quad O_5^A O_6^A O_5^B O_6^B$$

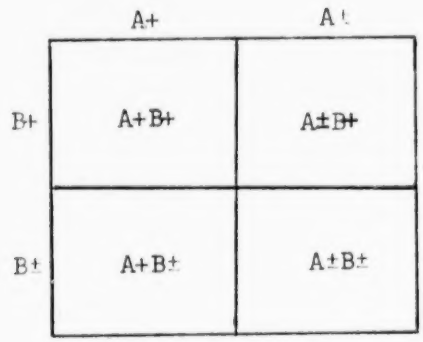
$$R_3 \quad O_1^A O_2^A O_1^B O_2^B \quad (X_3 \; O_3^A O_4^A O_3^B O_4^B) \quad O_5^A O_6^A O_5^B O_6^B$$

$$R_4 \quad O_1^A O_2^A O_1^B O_2^B \quad (X_4 \; O_3^A O_4^A O_3^B O_4^B) \quad O_5^A O_6^A O_5^B O_6^B$$

where R_1-R_4 are the four groups, X_1-X_4 are the four treatments and O_i^A and O_i^B are defined as follows:

O_1^A - PCA	O_3^A - TSIA	O_5^A - POA
O_2^A - PCSIA	O_4^A - TPIA	O_6^A - POSIA
O_1^B - PCB	O_3^B - TSIB	O_5^B - POB
O_2^B - PCSIB	O_4^B - TPIB	O_6^B - POSIB

Thus, given the 2 x 2 design matrix



The following hypotheses were tested: There is no significant interaction or main effects for levels of A and levels of B in:

I. Achievement: mean performance on

a) POA;
b) POB;

II. Time: mean total on

a) TSIA;
b) TSIB;
c) TPIA;
d) TPIB;
e) POSIA;
f) POSIB.

Analysis

The data were analyzed using the Clyde MANOVA program for a multivariate two-way analysis of covariance (Clyde, 1969). Because of the symmetry of the design the results for Concept B were viewed as a potential replication for the results for Concept A. Hence, the analysis for Concept B was done separately from the analysis for Concept A. Achievement variables were separated from time variables.

RESULTS

Pretest Analysis

Table IV summarizes the means and standard deviations for each cell on the pretest measures related to Concept A. The means of 6 out of a maximum of 10 indicate that subjects were reasonably skillful at calculations without parentheses for new binary operations.

TABLE IV

CELL MEANS AND STANDARD DEVIATIONS FOR PRETESTS RELATED TO CONCEPT A (PCA AND PSIA)

Cell	Treatment	Statistic	PCA	PSIA[a]
11	A+B+	M	6.063	185.000
		SD	2.816	97.175
12	A±B+	M	6.313	225.188
		SD	3.240	138.705
21	A+B±	M	5.500	230.688
		SD	2.338	109.842
22	A±B±	M	6.250	218.000
		SD	2.769	127.300

M - Mean, SD - Standard Deviation
[a]Stimulus intervals are in seconds.

The pretests PCA and PSIA were subject to multivariate and univariate analysis of variance to determine if significant differences existed between groups on measures related to Concept A before the treatments. Table V summarizes the results of the analysis. No p-values were less than .35.

TABLE V

MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE
OF PRETESTS RELATED TO CONCEPT A (PCA AND PSIA)

Variable(s)	Test	Source	df	F	p <
PCA, PSIA	M	A x B	2,59	0.413	.664
PCA	U		1,60	0.127	.723
PSIA	U		1,60	0.785	.379
PCA, PSIA	M	A	2,59	0.408	.667
PCA	U		1,60	0.507	.479
PSIA	U		1,60	0.212	.647
PCA, PSIA	M	B	2,59	0.267	.766
PCA	U		1,60	0.198	.658
PSIA	U		1,60	0.416	.521

M - Multivariate test, U - Univariate test

Table VI summarizes the means and standard deviations for each cell on the pretest measures related to Concept B. The means of 3 out of a maximum of 10 suggest that not all subjects were successful at calculations with parentheses for new binary operations. As on the pretests for Concept A, the subjects of cell 21 appears to be less skillful than the subjects of the other three cells.

TABLE VI

CELL MEANS AND STANDARD DEVIATIONS FOR
PRETESTS RELATED TO CONCEPT B (PCB AND PSIB)

Cell	Treatment	Statistic	PCB	PSIB[a]
11	A+B+	M	3.250	283.000
		SD	3.194	130.774
12	A+B+	M	3.688	298.625
		SD	3.301	128.094
21	A+B±	M	1.125	378.313
		SD	1.204	211.431
22	A+B!	M	2.875	275.250
		SD	2.986	122.181

M - Mean, SD - Standard Deviation
[a]Stimulus intervals are in seconds

The pretests PCB and PSIB were subjected to a multivariate and univariate analysis of variance to determine if significant differences existed between groups on measures related to Concept B before the treatments. Table VII summarizes the results of the analysis. While the multivariate tests were not significant, a univariate test on PCB was significant ($p < .05$).

TABLE VII

MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE OF PRETESTS RELATED TO CONCEPT B (PCB AND PSIB)

Variable(s)	Test	Source	df	F	p <
PCB, PSIB	M	A x B	2,59	1.481	.236
PCB	U		1,60	0.876	.353
PSIB	U		1,60	2.420	.125
PCB, PSIB	M	A	2,59	1.669	.197
PCB	U		1,60	2.433	.124
PSIB	U		1,60	1.313	.256
PCB, PSIB	M	B	2,59	2.410	.099
PCB	U		1,60	4.388	.040*
PSIB	U		1,60	0.889	.350

M - Multivariate test, U - Univariate test
*$p < .05$

Because the pretests appeared to be fairly reliable ($r > .80$), the groups appeared to differ on at least one of the pretests, and not all subjects were successful on the pretests, covariance procedures were chosen for the analysis.

Achievement, Concept A (POA)

Table VIII gives the unadjusted means and standard deviations for the posttest related to Concept A (POA).

TABLE VIII

CELL MEANS AND STANDARD DEVIATIONS FOR
POSTTEST RELATED TO CONCEPT A (POA)

Cell	Treatment	Statistic	POA
11	A+B+	M	11.875
		SD	2.729
12	A±B+	M	12.563
		SD	3.444
21	A+B±	M	12.063
		SD	2.594
22	A±B±	M	13.313
		SD	2.056

M - Mean, SD - Standard Deviation

The variable POA, as the major criterion variable for Concept A, was analyzed using a univariate analysis of covariance with PCA as covariate. The use of PCB as a covariate was rejected as no calculations with parentheses are required to classify the instances of POA. Table IX summarizes the results of the analysis of POA. While the regression was significant ($p < .004$) no p-values for interaction or main effects were less than .2.

TABLE IX

ANALYSIS OF COVARIANCE FOR CONCEPT A OF POA USING PCA AS COVARIATE

Source	df	F	p <
Equality of Regression	3,56	0.310	.818
Regression	1,59	8.987	.004**
A x B	1,59	0.089	.767
A	1,59	1.485	.228
B	1,59	0.803	.374

POA - Posttest for Concept A.
PCA - Pretest, Calculations without parentheses.
**p < .01

Achievement, Concept B (POB)

Table X gives the cell means and standard deviations for the post-test for Concept B (POB).

TABLE X

CELL MEANS AND STANDARD DEVIATIONS FOR POSTTEST RELATED TO CONCEPT B (POB)

Cell	Treatment	Statistic	POB
11	A+B+	M	8.688
		SD	2.726
12	A±B+	M	9.125
		SD	2.247
21	A+B±	M	8.750
		SD	2.236
22	A±B±	M	11.625
		SD	2.705

M - Mean, SD - Standard Deviation

The variable POB, as the major criterion variable for Concept B, was analyzed using a univariate analysis of covariance with PCA and PCB as covariates. Both PCA and PCB were used as covariates as the calculations for POB required both calculations with and without parentheses. Table XI summarizes the results of the analysis of POB. Both the A effect and B effect were significant ($p < .025$).

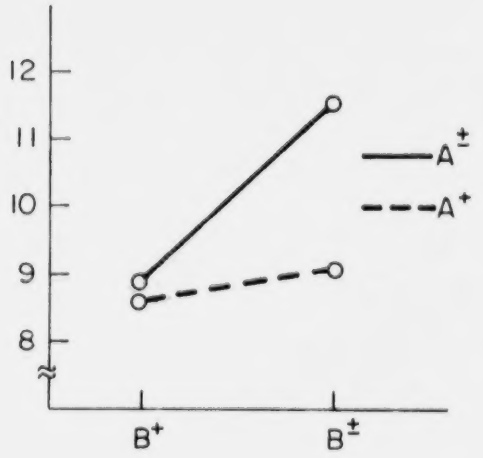
TABLE XI

ANALYSIS OF COVARIANCE FOR CONCEPT B OF POB USING PCA AND PCB AS COVARIATES

Source	df	F	p <
Equality of Regression	6,52	7.351	.280
Regression	1,58	2.615	.082
A x B	1,58	3.195	.079
A	1,58	5.356	.024*
B	1,58	5.848	.019*

POB - Posttest for Concept B
PCA - Pretest, Calculations without parentheses
PCB - Pretest, Calculations with parentheses
*p < .05

Figure 4 shows the adjusted cell and margin means and a plot of the cell means. Most of the difference appears to be accounted for by the A±B± treatment.



	A^+	$A^\pm$	
B^+	8.614	8.954	8.784
$B^\pm$	9.048	11.571	10.310
	8.831	10.263	

Figure 4. Adjusted means for POB (A effect, B effect)

The B effect suggests that treatment B± is more effective than treatment B+. The A effect suggests that A± is more effective than A+. That is, in addition to an effect for negative instances on Concept B, there appears to be a transfer effect from the treatment of negative instances for Concept A to achievement for Concept B.

Time, Concept A (PCSIA, TSIA, TPIA, POSIA)

Table XII summarizes the means and standard deviations for each cell on the time variables for Concept A.

TABLE XII

CELL MEANS AND STANDARD DEVIATIONS FOR TIME VARIABLES[a] RELATED TO CONCEPT A (PCSIA, TSIA, TPIA, AND POSIA)

Cell	Treatment	Statistic	PCSIA	TSIA	TPIA	POSIA
11	A+B+	M	185.000	264.938	49.625	138.188
		SD	97.175	225.804	23.180	100.702
12	A+B±	M	225.188	302.313	63.250	223.188
		SD	138.705	143.794	31.634	97.718
21	A±B+	M	230.688	277.875	63.625	166.625
		SD	109.842	159.595	43.297	66.635
22	A±B±	M	218.000	300.625	44.063	179.000
		SD	127.300	148.527	19.999	76.522

M - Mean, SD - Standard Deviation
PCSIA - Pretest, Calculations without parentheses Stimulus Interval.
TSIA - Treatment Stimulus Interval for Concept A.
TPIA - Treatment Postfeedback Interval for Concept A.
POSIA - Posttest Stimulus Interval for Concept A.
[a]All time variables are in seconds.

The time variables were subjected to a multivariate analysis of covariance. Appendix A.9 reports the correlation matrix for all variables of the study. The significant correlations ($p < .05$) among some of the time variables support the use of multivariate analysis. (See Table XVII, Appendix A.9). The total stimulus interval on the pretest (PCSIA) was used as the covariate. Table XII summarizes the results of the analysis of the time variables for Concept A. The only multivariate F which was significant was for the regression effect ($p < .001$). Two univariate regression effects were also significant ($p < .001$ and $p < .01$), TSIA and POSIA. According to Cramer and Bock

TABLE XIII

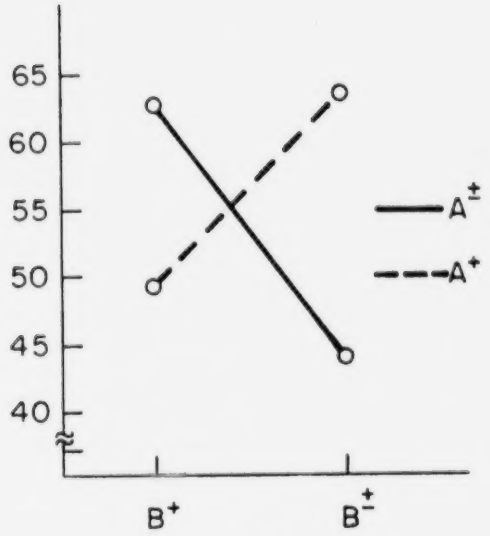
MULTIVARIATE AND UNIVARIATE ANALYSIS OF COVARIANCE FOR TIME VARIABLES RELATED TO CONCEPT A (TSIA, TPIA, AND POSIA) USING PCSIA AS COVARIATE

Variable(s)	Test	Source	df	F	p <
TSIA, TPIA, POSIA	M	Equality of	9,131.6	1.602	.121
TSIA	U	Regression	3,56	1.792	.159
TPIA	U		3,56	2.139	.107
POSIA	U		3,56	2.271	.090
TSIA, TPIA, POSIA	M	Regression	3,57	6.880	.001**
TSIA	U		1,59	20.898	.001**
TPIA	U		1,59	0.000	.986
POSIA	U		1,59	9.037	.004**
TSIA, TPIA, POSIA	M	A x B	3,57	2.532	.066
TSIA	U		1,59	0.106	.746
TPIA	U		1,59	4.495	.038*
POSIA	U		1,59	2.054	.157
TSIA, TPIA, POSIA	M	A	3,57	2.461	.072
TSIA	U		1,59	0.282	.597
TPIA	U		1,59	0.144	.706
POSIA	U		1,59	4.895	.031*
TSIA, TPIA, POSIA	M	B	3,57	0.158	.924
TSIA	U		1,59	0.053	.819
TPIA	U		1,59	0.109	.742
POSIA	U		1,59	0.404	.527

*p < .05, **p < .01

(1966), if the multivariate test is significant, then the univariate tests can be examined with some assurance that significant effects exist. If we demand multivariate significance ($p < .05$) before examining the univariate test, no other differences were significant. Some researchers have adopted the strategy of examining the univariate tests at the .05 level if the multivariate test is significant at the .10 level (Walberg, Sorenson, and Fischbach, 1972).

Pointing out that this strategy increases the probability of making a Type I error, we tentatively observe that the p-values for the multivariate tests for A x B and A are less than .08. The underlying univariate test of A x B for TPIA is significant ($p < .05$). Figure 5 gives the adjusted means for TPIA and displays a plot of the

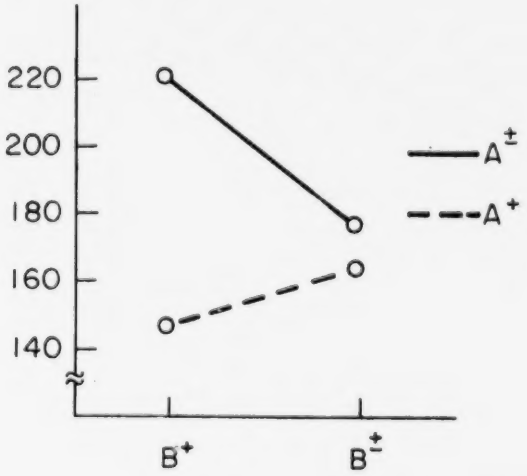


	A^{+}	$A^{\pm}$	
B^{+}	49.607	63.256	56.432
$B^{\pm}$	63.635	44.064	53.850
	56.621	53.660	

Figure 5. Adjusted means for TPIA (A x B effect)

interaction. We simply observe that, if the interaction is significant, the significance probably lies with variable TPIA. It appears that the interaction is a disordinal interaction.

The underlying univariate test of A for POSIA is significant ($p < .05$). Figure 6 gives the adjusted means for POSIA and a plot of the cell means. We observe that, if the A effect is significant, the significance probably lies with variable POSIA. It would appear that the introduction of negative instances in the treatment for Concept A caused subjects to spend more time responding to the items of the post-test for Concept A.



	A^{+}	$A^{\pm}$	
B^{+}	146.047	220.419	183.233
$B^{\pm}$	162.402	178.132	170.267
	154.224	199.276	

Figure 6. Adjusted means for POSIA (A effect)

Time, Concept B (PCSIB, TSIB, TPIB, and POSIB)

Table XIV summarizes the means and standard deviations for each cell on the time variables for Concept B.

TABLE XIV

CELL MEANS AND STANDARD DEVIATIONS FOR TIME VARIABLES[a] RELATED TO CONCEPT B (PCSIB, TSIB, TPIB, AND POSIB)

Cell	Treatment	Statistic	PCSIB	TSIB	TPIB	POSIB
11	A+B+	M	283.000	245.250	48.000	285.750
		SD	130.774	201.570	20.594	332.169
12	A+B+	M	298.625	427.125	58.813	304.375
		SD	128.094	228.676	33.837	130.443
21	A+B+	M	378.313	348.438	68.938	280.313
		SD	211.431	216.302	45.414	161.453
22	A+B+	M	275.250	373.813	49.500	266.375
		SD	122.181	171.344	32.656	169.930

M - Mean, SD - Standard Deviation
PCSIB - Pretest, Calculations with parentheses Stimulus Interval.
TSIB - Treatment Stimulus Interval for Concept B.
TPIB - Treatment Postfeedback Interval for Concept B.
POSIB - Posttest Stimulus Interval for Concept B.
[a]All time variables are in seconds.

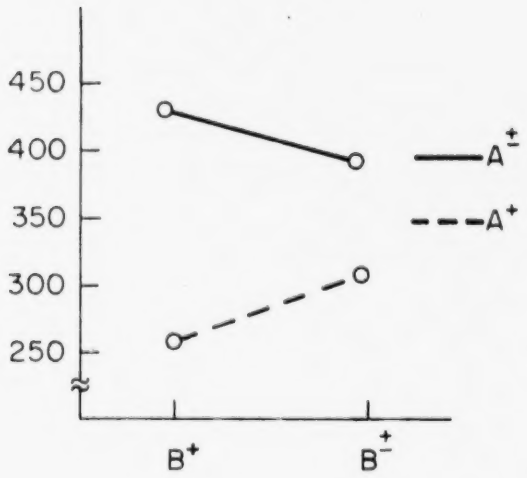
As in the Concept A analysis, the time variables were subjected to a multivariate analysis of covariance. The total stimulus interval on the pretest (PCSIB) was used as the covariate. Table XV summarizes the results of the analysis of the time variables for Concept B. Consistent with the analysis for Concept A, the multivariate regression effect was significant ($p < .01$) as were the underlying univariate tests for TSIB and POSIA ($p < .01$).

TABLE XV

MULTIVARIATE AND UNIVARIATE ANALYSIS OF COVARIANCE FOR TIME VARIABLES RELATED TO CONCEPT B (TSIB, TPIB, AND POSIB) USING PCSIB AS COVARIATE

Variable(s)	Test	Source	df	F	p <
TSIB, TPIB, POSIB	M	Equality of	9,131.6	0.633	.768
TSIB	U	Regression	3,56	0.685	.565
TPIB	U		3,56	0.804	.497
POSIB	U		3,56	0.026	.994
TSIB, TPIB, POSIB	M	Regression	3,57	4.534	.006**
TSIB	U		1,59	10.226	.002**
TPIB	U		1,59	1.858	.178
POSIB	U		1,59	7.153	.010**
TSIB, TPIB, POSIB	M	A x B	3,57	1.364	.263
TSIB	U		1,59	0.948	.334
TPIB	U		1,59	2.175	.146
POSIB	U		1,59	0.045	.832
TSIB, TPIB, POSIB	M	A	3,57	2.650	.057
TSIB	U		1,59	6.814	.011*
TPIB	U		1,59	0.091	.764
POSIB	U		1,49	0.191	.664
TSIB, TPIB, POSIB	M	B	3,57	0.409	.747
TSIB	U		1,59	0.017	.896
TPIB	U		1,59	0.264	.609
POSIB	U		1,59	0.559	.458

*p < .05, **p < .01



	A^{+}	$A^{\pm}$	
B^{+}	258.608	432.392	345.500
$B^{\pm}$	312.441	391.184	351.812
	285.524	44.788	

Figure 7. Adjusted means for TSIB (A effect)

No other multivariate tests were significant. The p-value for the multivariate test of the A effect was less than .06. The underlying univariate test for the A effect on TSIB was significant ($p < .025$). Figure 7 gives the adjusted means for TSIB and displays a plot of the cell means. We observe that if the A effect is significant, the significance probably lies with variable TSIB. It would appear that the introduction of negative instances in the treatment for Concept A caused subjects to spend more time responding to the treatment items for Concept B. Perhaps this is further evidence of the transfer effect of negative instances for Concept A to performance on Concept B.

Summary

Table XVI summarizes the results of the analyses for Concept A and Concept B. For Concept B, there was a significant A effect and B effect on achievement (POB) favoring treatments of both positive and negative instances.

TABLE XVI

SUMMARY OF ANALYSES FOR CONCEPT A AND CONCEPT B FOR WHICH STATISTICALLY SIGNIFICANT DIFFERENCES WERE FOUND ($p < .05$)

Variable(s)	Effects	$p <$	Level Favored +	Level Favored ±	Transfer Effect
Concept A					
TPIA	A x B	.038[a]			Yes
POSIA	A	.031[a]		Yes	
Concept B					
POB	A	.024		Yes	Yes
	B	.019		Yes	
TSIB	A	.011[a]		Yes	Yes

[a]Univariate tests in a three variable multivariate analysis where the multivariate p-values were .066, .072, and .056, respectively.

Risking a possible multivariate Type I error ($p < .075$) there appears to be an interaction effect on postfeedback interval during the treatment for Concept A (TPIA) and an A effect on stimulus interval during the posttest for Concept A (POSIA) and the treatment for Concept B (TSIB). The treatments of positive and negative instances appeared to have increased the stimulus interval times.

CONCLUSIONS

Two questions were examined:

1. What are the different effects of an instructional sequence of positive and negative instances and a sequence of all positive instances in the acquisition of commutativity and/or associativity?

2. Assuming there are effects for negative instances, do the effects of negative instances for one concept transfer to another concept?

Question 1 was answered as follows:

For the acquisition of the concept of associativity, a sequence of positive and negative instances was favored over a sequence of all positive instances.

Question 2 was answered as follows:

The acquisition of the concept of associativity was improved by the effect of negative instances for commutativity. Transfer occurred.

Discussion

As was pointed out in the results, if one relaxes the conditions controlling Type I error, it appeared that negative instances of commutativity increased the total stimulus interval for instances of both associativity and commutativity. This result would tend to weaken the advantage cited for negative instances in the conclusions. Further study and control of the time variables appears to be appropriate.

Another possible alternative for the results cited can be proposed. It is possible that the subjects simply preserved the same proportion of positive and negative instances during the posttest as was presented to them during treatment, regardless of the concept involved. The low reliability estimates reported for the posttests may suppor this hypothesis. Mandler and Cowan (1962) have reported such results. An exploratory analysis was performed on the data of this experiment examining the mean number of yes responses for each cell on the posttests. The results, reported in Appendix A.10 show both an A and B effect on both the criteria for Concept A and for Concept B ($p < .01$). The more negative instances presented in the treatment, the fewer yes responses were given on the posttests. These data also support the conclusion that the treatments did have an effect. Bourne has also suggested that the

variable of the proportion of positive instances is important and should be investigated further (Bourne and Guy, 1968; Bourne, 1972).

While the subjects were old enough to be in the formal reasoning stage (Inhelder and Piaget, 1958) and on other concept learning tasks, ninth graders appear to be as successful as college students (Bourne and O'Banion, 1971), it is possible that the results would be different for subjects of an older age.

In summary, it is clear that negative instances do have an effect on concept learning. However, the underlying explanation for the effects is not clear from these data.

RECOMMENDATIONS

Most teachers of mathematics are familiar with the problem of students overgeneralizing concepts in mathematics. The following examples are familiar to all mathematics teachers:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{x + y}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$$

$$(a + b)^n = a^n + b^n$$

$$\int fg = \int f \int g$$

$$f \circ g = g \circ f$$

One possible strategy for discouraging such overgeneralizations could be the introduction of negative instances during the teaching of each concept. There has been some evidence that such a strategy is effective (Shumway, 1971; Tennyson, Woolley, and Merrill, 1971). There has also been a great deal of evidence from experimental psychology to support the exclusion of negative instances in concept learning (Clark, 1971).

The results of this study are consistent with Tennyson, Woolley, and Merrill (1971) and Shumway (1971). A treatment of both positive and negative instances was superior to a treatment of positive instances alone and the effect of negative instances appeared to transfer from one concept to another. Three possible explanations for the results have been proposed:

E1. Negative instances are a necessary and integral part of the learning of each concept;

E2. Negative instances teach subjects to be skeptical, independent of a given concept;

E3. Negative instances simply teach subjects the proportion of the criterion instances which should be classified as negative using guessing.

None of these possible explanations (E1-E3) can be rejected or accepted based on the results of this study alone. The following recommendations are suggested for further study:

S1. A replication of the current study with different concepts;

S2. A replication of the current study with subjects of an older age;

S3. A replication of the current study with subjects of an older age and with a manipulation of the proportion of negative instances in the criterion measure.

These studies are not recommended simply to verify or not verify that negative instances promote concept acquisition. In order to use negative instances effectively it is necessary to know why negative instances make a difference, as well as that negative instances do make a difference.

As a practical recommendation to the teacher of concepts in schools the following seems appropriate:

> There is some evidence that negative instances are important in the learning of concepts. It would seem appropriate for the classroom practitioner to begin to cautiously introduce negative instances of concepts in instruction.

CITED REFERENCES

Ausubel, D. P. Educational psychology: a cognitive view. New York: Holt, Rinehart, and Winston, 1968.

Bereiter, C., and Engelmann, S. Teaching disadvantaged children in the preschool. Englewood Cliffs, New Jersey: Prentice-Hall, 1966, 48-49.

Bourne, Lyle E. Jr. Human conceptual behavior. Boston: Allyn and Bacon, Inc., 1966.

Bourne, Lyle E. Jr. Learning and utilization of conceptual rules. Publication No. 89 of the Institute of Behavioral Science, University of Colorado, 1967, 1-32.

Bourne, L. E., Jr. personal communication, May, 1972.

Bourne, Lyle E. Jr., and Dominowski, Roger L. Thinking. Annual Review of Psychology 1972, 105-130.

Bourne, L. E., Jr, Ekstrand, B. R., and Montgomery, B. Concept learning as a function of the conceptual rule and the availability of positive and negative instances. Journal of Experimental Psychology, 1969, 82, 538-544.

Bourne, Lyle E. Jr., and Guy, Donald E. Learning conceptual rules: I. some interrule transfer effects. Journal of Experimental Psychology, 1968, 76, 423-429.

Bourne, Lyle E. Jr., and Guy, Donald E. Learning conceptual rules. II. the role of positive and negative instances. Journal of Experimental Psychology, 1968, 77, 488-494.

Bourne, L. E. Jr., and O'Banion, Katy. Conceptual rule learning and chronological age. Developmental Psychology, 1971, 5, 525-534.

Braley, Loy S. Strategy selection and negative instances in concept learning. Journal of Educational Psychology, 54, 1963, 154-159.

Bruner, J. S., Goodnow, J. J., and Austin, G. A. A study of thinking. New York: Wiley, 1956.

Chlebek, Judith, and Dominowski, Roger L. The effect of practice on utilization of information from positive and negative instances in identifying disjunctive concepts. Canadian Journal of Psychology, 1970, 24, 64-69.

Clark, D. Cecil. Teaching concepts in the classroom: a set of teaching prescriptions derived from experimental research. Journal of Educational Psychology, 1971, 62, 253-278.

Clyde, Dean J. Multivariate analysis of variance on large computers. Miami: Clyde Computing Service, 1969.

Conant, Michael Bradford. Or: the utilization of negative instances in disjunctive concept learning. (Doctoral dissertation, Stanford University) Ann Arbor, Mich.: University Microfilms, 1965, No. 66-2538.

Conant, Michael B , and Trabasso, Tom. Conjunctive and disjunctive concept formation under equal-information conditions. Journal of Experimental Psychology, 1964, 67, 250-255.

Cramer, Elliot M., and Bock, R. Darrel. Multivariate analysis. Review of Educational Research, 1966, 36, 604-617.

Cronback, Lee J. The two disciplines of scientific psychology. The American Psychologist, 1957, 12, 671-684.

Davidson, Michael. Positive versus negative instances in concept identification problems matched for logical complexity of solution procedures. Journal of Experimental Psychology, 1969, 80, 369-373.

Denney, Nancy Wadsworth. The effect of varying percentage of positive and negative instances and instructions on conjunctive and disjunctive learning. Psychonomic Science 1969, 19, 193-194.

Denny J. Peter, and Benjafield, John G. Concept identification strategies used for positive and negative instances. Psychonomic Science, 1969, 14, 277-280.

Dervin, Dennis, and Deffenbacher, Kenneth. Effects of proportion of positive instances and degree of restriction on the induction of a principle. Psychonomic Science, 1970, 31, 79-80.

Dienes, Z. P. The power of mathematics. London: Hutchinson Educational LTD., 1964.

Dominowski, Roger L Stimulus memory in concept attainment. Psychonomic Science, 1968, 0, 359-360.

Donaldson, Margaret. Positive and negative information in matching problems. British Journal of Psychology, 1959, 50, 253-262.

Dossey, J. A. The relative effectiveness of four strategies for teaching disjunctive concepts in mathematics. A paper presented at the annual meeting of the American Educational Research Association, Chicago, April, 1972.

Duncan, Carl P. Induction of a principle. Quarterly Journal of Experimental Psychology, 1964, 16, 373-377.

Eifermann, Rivka R., and Steinitz, Ruth. A comparison of conjunctive and disjunctive concept identification. The Journal of General Psychology, 1971, 85, 29-37.

Frase, Lawrence T. Concept learning and learning from prose. A paper presented at the annual meeting of the American Educational Research Association, Chicago, April, 1972.

Fraunfelker, George Herbert. Attribute identification in concept learning as a function of concept type, mode of presentation and positive versus negative initial instances. DAI, 1971, 786-A-787-A.

Freibergs, Vaira, and Tulving, Endel. The effect of practice on utilization of information from positive and negative instances in concept identification. Canadian Journal of Psychology, 1961, 15, 101-106.

Fryatt, Maurice J., and Tulving, Endel. Interproblem transfer in identification of concepts involving positive and negative instances. Canadian Journal of Psychology, 1963, 17, 106-117.

Gagne, R. M. The conditions of learning. New York: Holt, Rinehart, and Winston, 1965.

Gelbaum, B. R., and Olmsted, J. M. H. Counterexamples in analysis. Holden-Day, Inc., 1964.

Giambra, Leonard M. Effect of number of irrelevant dimensions with ten concept types on the attribute identification task in the selection mode with exemplar and non-exemplar start cards. Psychonomic Science, 1969, 14, 75-76.

Glanzer, Murray, Huttenlocher, Janellen, and Clark, William H. Systematic operations in solving concept problems: a parametric study of a class of problems. Psychological Monographs, 1963, 77, 1-29.

Gough, Phillip B. Grammatical transformations and speed of understanding. Journal of Verbal Learning and Verbal Behavior, 1965, 4, 107-111.

Haygood, Robert C., and Devine James V. Effects of composition of the positive category on concept learning. Journal of Experimental Psychology, 1967, 74, 230-235.

Haygood, Robert C., and Stevenson, Michael. Effects of proportion of positive instances upon concept learning. Psychological Reports, 1967, 20, 179-182.

Hovland, C. I. A "communication analysis" of concept learning. Psychological Review, 1952, 59, 461-472.

Hovland, Carl I., and Weiss, Walter. Transmission of information concerning concepts through positive and negative instances. Journal of Experimental Psychology, 1953, 45, 175-182.

Hunt, E. B. Concept learning: an information processing problem. New York: Wiley, 1962.

Hunt, Earl B., and Hovland, Carl I. Order of consideration of different types of concepts. Journal of Experimental Psychology, 1960, 59, 220-225.

Huttenlocher, Janellen. Some effects of negative instances on the formation of simple concepts. Psychological Reports, 1962, 4, 35-42.

Huttenlocher, Janellen. How certain reasoning problems are solved. Journal of Verbal Learning and Verbal Behavior, 1967, 6, 802-808.

IBM, Coursewriter III for system/360, Author's Guide. White Plains: IBM, 1969.

Inhelder, B., and Piaget, J. The growth of logical thinking. New York: Basic Books, Inc., 1968.

Johnson-Laird, P. N., and Tagart, Joanna. How implication is understood. American Journal of Psychology, 1969, 82, 367-373.

Kendler, Howard H. The concept of the concept. Categories of human learning. New York: Academic Press, 1964.

Kendler, T. S. Concept formation. Annual Review of Psychology, 1961, 12, 447-472.

Krebs, Marjorie J., and Lovelace, Eugene A. Disjunctive concept identification: Stimulus complexity and positive versus negative instances. Journal of Verbal Learning and Verbal Behavior, 1970, 96, 653-657.

Mandler, George, and Cowan, Philip A. Learning of simple structures. Journal of Experimental Psychology, 1962, 64, 177-183.

Marine, J. Efficacy of nonexample moves in teaching mathematical concepts. A paper presented at the annual meeting of the American Educational Research Association, Chicago, April, 1972.

Markle, S. M., and Tiemann, P. W. Conceptual learning and instructional design. Journal of Educational Technology, 1970, 1, 52-62.

Markle, S. M., and Tiemann, P W. Some principles of instructional design at higher cognitive levels. Invited address to Lehrsysteme 72, International Congress for Instructional Technology, Berlin, Germany, April 5-8, 1972.

Mayzner, M. S. Verbal concept attainment: a function of the number of positive and negative instances presented. Journal of Experimental Psychology, 1962, 63, 314-319.

Merriman, H. O. The Columbus School Profile, Columbus: Columbus Public Schools, 1969.

O'Neill, Andrea Seiver. The effect of positive and negative information upon conjunctive concept learning at varying levels of task complexity and concept size. DAI, 1969, 30, 3112-B.

Pereboom, A. C. Some fundamental problems in experimental psychology: an overview. Psychological Reports, 1971, 28, 439-455.

Schvaneveldt, Roger W. Concept identification as a function of probability of positive instances and number of relevant dimensions. Journal of Experimental Psychology, 1966, 72, 649-654.

Shore, Eugene, and Sechrest, Lee. Concept attainment as a function of number of positive instances presented. Journal of Educational Psychology, 1961, 52, 303-307.

Shumway, R. J. The role of counterexamples in the development of mathematical concepts of eighth grade mathematics students. DAI, 1970, 31, 3368A-3369A.

Shumway, Richard J. Negative instances and mathematical concept formation: a preliminary study. Journal for Research in Mathematics Education, 1971, 2, 218-277.

Smoke, Kenneth L. Negative instances in concept learning. Journal of Experimental Psychology, 1933, 16, 583-588.

Steen, L. A., and Seebach, J. A., Jr. Counterexamples in topology. New York: Holt, Rinehart, and Winston. 1971.

Tagatz, Glenn E., Meinke, Dean L., and Lemke, Elmer A. Developmental aspects relative to information processing and concept attainment. The Journal of Genetic Psychology, 1968, 113, 253-262.

Taplin, John E. Effect of initial instance on attribute identification of concepts using a selection procedure. Journal of Experimental Psychology, 1971, 88, 177-181.

Tavrow, Wendy Newfield. The effects of practice and of previous familiarity with the universe in forming concepts from negative instances. DAI, 1966, 26, 4857-4858.

Tennyson, Robert D., Woolley, F. Ross, and Merrill, M. David. Exemplear and nonexemplar variables which produce correct concept classification behavior and specified classification errors. A paper presented at the annual meeting of the American Educational Research Association, New York, February, 1971.

Walberg, H. J., Sorenson, J., and Fischbach, T. Ecological correlates of ambience in the learning environment. American Educational Research Journal, 1972, 9, 139-148.

Wales, R. J., and Grieve, R. What is so difficult about negation? Perception & Psychophysics, 1969, 6, 327-332.

Wason, P. C. The processing of positive and negative information. Quarterly Journal of Experimental Psychology, 1959, 11, 92-107.

Wason, P. C. On the failure to eliminate hypotheses in a conceptual task. The Quarterly Journal of Experimental Psychology, 1960, 12, 129-140.

Weber, Robert, and Woodward, Addison Jr. Transformations of positive and negative information in a modified learning-set task. Journal of Experimental Psychology, 1966, 72, 492-496.

Wickelgren, Wayne A., and Cohen, David H. An artificial language and memory approach to concept attainment. Psychological Reports, 1962, 10, 815-827.

Wojtaszek, Florence Patricia. Concept formation as a function of information type and rule difficulty. DAI, 1971, 6310-B-6311-B.

APPENDIXES

A.1 UNCITED REFERENCES

Adams, J. A. Multiple versus single problem training in human problem solving. Journal of Experimental Psychology, 1954, 48, 15-18.

Bourne, Lyle E. Jr. Knowing and using concepts. Psychological Review, 1970, 77(6), 545-556.

Bourne, Lyle E. Jr., Guy, D. E., Dodd, David H. and Justesen, Don R. Concept identification: the effects of varying length and informational components of the intertrial interval. Journal of Experimental Psychology, 1965, 69, 624-629.

Brinley, Joseph F., and Sardello, Robert J. Increasing the utility of negative instances in conjunctive concept identification. Psychonomic Science, 1970, 18, 101-102.

Callantine, Mary F. and Warren J. M. Learning sets in human concept formation. Psychological Reports, 1955, 1, 363-367.

Finney, D. J. Multivariate analysis of agricultural experiments. Biometrics, 1956, 12, 67-71.

Gabriel, K. R. Simultaneous test procedures in multivariate analysis of variance. Biometrika, 1968, 55, 489-504.

Giambra, Leonard M. Conditional and biconditional rule difficulty with attribute identification, rule learning, and complete learning task. Journal of Experimental Psychology, 1970, 86, 250-254.

Glass, G. V. and Stanley, J. C. Statistical methods in education and psychology. Endlewood Cliffs: Prentice-Hall, 1970.

Greene, Judith M., and Wason, P. C. Negation: a rejoinder to Wales and Grieve. Perception & Psychophysics, 1970, 8, 238-239.

Haygood, Robert C., and Bourne, Lyle E. Jr. Attribute- and rule-learning aspects of conceptual behavior. Psychological Review, 1965, 72, 175-195.

Hays, W. L. Statistics for psychologists. New York: Holt, Rinehart, and Winston, 1963.

Holland, James G. Discussion of papers by Lyle E. Bourne and Edwin J. Martin. In B. Kleinmuntz (ed.) Memory and the Structure of Concepts. New York: Wiley, 1967.

Houde, Richard A. A prospectus in mathematics education. An experimental study. A dissertation proposal, University of Tennessee, 1971.

Houtz, J. C , Moore, J. W., and Davis, J. K. The effects of different types of positive and negative instances in concept learning. Paper presented at the annual meeting of the American Education Research Association, Chicago, April, 1972.

Johnson-Laird, P. N., and Wason, P. C. Insight into a logical relation. Quarterly Journal of Experimental Psychology, 1970, 49-61.

Johnson-Laird, P. N., and Wason, P. C. A theoretical analysis of insight into a reasoning task. Cognitive Psychology, 1970, 1, 134-148

Johnson, Peder J., and White, Roger H. Jr. Effects of pretraining and stimulus composition on rule learning. Journal of Experimental Psychology, 1969, 80, 450-454.

Jones, Sheila. The effect of a negative qualifier in an instruction. Journal of Verbal Learning and Verbal Behavior, 1966, 5, 497-501.

Kates, Solis L., and Yudin, Lee. Concept attainment and memory. Journal of Educational Psychology, 1964, 55, 103-109.

Knowles, Brian A., Hammond, Kenneth R., Stewart, Thomas R., and Summers, David A. Positive and negative redundancy in multiple cue probability tasks. Journal of Experimental Psychology, 1970, 90, 158-159.

Laughlin, Patrick R. Information specification in the attention of conditional concepts. Journal of Experimental Psychology, 1969, 79, 370-372.

Laughlin, Patrick R. Concept attainment as a function of information specification and concept complexity. Journal of Experimental Psychology, 1971, 90, 334-343.

Martin, E. Formation of concepts in B. Kleinmuntz (ed.), Memory and the structure of concepts. New York: Wiley, 1967.

Merrill, M. David, and Tennyson, Robert D. Attribute prompting variables in classroom concepts. Department of Instructional Research and Development Division of Instructional Services-Brigham Young University, 1971, 28 pages.

Merrill, M. David, and Tennyson, Robert D. Concept acquisition and specified errors as a function of relationships between positive and negative instances. Department of Instructional Research and Development Division of Instructional Services-Brigham Young University, 1971, 32 pages.

Murdock, B. Discussion of papers by Lee W. Gregg & Earl Hunt. In B. Kleinmuntz (ed.), Memory and the structure of concepts. New York: Wiley, 1967.

Nahinsky, Irwin D. A test of axioms of all- or-none concept identification models. Journal of Verbal Learning and Verbal Behavior, 1968, 7, 593-601.

Neisser, Ulric, and Weene, Paul. Hierarchies in concept attainment. Journal of Experimental Psychology, 1962, 64, 640-645.

Newell, A., and Simon H. Overview: memory and process in concept formation, in B. Kleinmuntz(ed.) Memory and the structure of concepts. New York: Wiley, 1967.

Pishkin, Vladimir, and Wolfgang, Aaron. Number and type of available instances in concept learning. Journal of Experimental Psychology, 1965, 69, 5-8.

Pruzek, R. M. Methods and problems in the analysis of multivariate data. Review of Educational Research, 1971, 41, 163-190.

Tennyson, R. D. Correct classification, overgeneralization, undergeneralization, and misconception as a function of probability, divergence and matching of instances in concept acquisition. Working Paper No. 19, Department of Instructional Research and Development. Brigham Young University, 1971.

Tennyson, Robert D. Instructional variables that predict specified learner concept acquisition and errors. Reprinted from the Proceedings, 79th Annual Convention, APA, 1971, 551-552.

Trabasso, T. Verification of conceptual rules. Proceedings, 75th Annual Convention, APA, 1967, 41-42.

Van De Geer, John P., and Jaspars, Joseph M. F. Cognitive functions. Annual Review of Psychology, 1963, 145-176.

Wallace, John Jr., and Sechrest, Lee. Relative difficulty of conjunctive and disjunctive concepts. Journal of Psychological Studies, 1961, 12, 97-104.

Walters, Stephen B., Schmidt, Stephen W., Bornstein, Robert, and White, Raymond M. Jr. Effects of pre-task frequency and conjunctive rule information on concept identification. Journal of Experimental Psychology, 1971, 89, 351-356.

Wason, P. C. Regression in reasoning? British Journal of Psychology, 1969, 4, 471-480.

Wells, Herbert. Facilitation of concept learning by a "simultaneous contrast" procedure. Psychonomic Science, 1967, 9, 609-610.

Wells, Herbert, and Watson, Daniel. Strategy training and practice in disjunctive concept attainment. Psychological Reports, 1965, 17, 925-926.

Wetherick, N. E. On the representativeness of some experiments in cognition. Bulletin of the British Psychological Society, 1970, 23, 213-214.

Whitfield, J. W. An experiment in problem solving. Quarterly Journal of Experimental Psychology, 1951, 3, 184-197.

Winer, B. J. Statistical principles in experimental design. New York: McGraw-Hill, 1962.

Woolley, F. Ross, and Tennyson, Robert D. Conceptual model of classification behavior. Department of Instructional Research and Development Division of Instructional Services-Brigham Young University, 1970, 9 pages.

A.2 GLOSSARY OF TERMS

1. Associative: A binary operation * on a set S is said to be associative if and only if for every a, b, and c in S,
   $a * (b * c) = (a * b) * c$.

2. Attributes: Discernable characteristics of an object, event, or idea that distinguish it from other objects, events, or ideas.

3. Bioconditional: A statement is a bioconditional if and only if it is of the form "_____ if and only if _____."

4. Binary operation: A binary operation * on a set S is a correspondence which associates with every ordered pair (a,b) of elements of S a unique element a * b of S.

5. Commutative: A binary operation * on a set S is said to be commutative if and only if for every a and b in S,
   $a * b = b * a$.

6. Concept: A concept over the class X is a partitioning of a class X into two disjoint classes X_1 and X_2.

7. Concept acquisition: Concept acquisition tasks are those concept learning tasks where a simple set of instructions would not produce the same behavior as the conventional training procedures. (Kendler, H. H., 1964, p. 227)

8. Concept identification: Concept identification tasks are those concept learning tasks where instructions could produce the same behaviors as the conventional training procedures. (Kendler, H. H., 1964, p. 227)

9. Conditional: A statement is said to be a conditional if and only if it is of the form "If _____, then _____."

10. Conjunction: A statement is said to be a conjunction if and only if it is of the form "_____ and _____."

11. Coursewriter III: Coursewriter III is an interactive computer language designed for computer-assisted instruction (CAI).

12. Delay of informative feedback: The delay of informative feedback is the length of time between the subject's response to a question and the presentation of the feedback associated with the subject's response.

13. Disjunction: A statement is said to be a disjunction if and only if it is of the form "_____ and/or _____."

14. Negative instance: Given a concept over a class X with partition X_1, X_2 and given that the elements of X_1 are examples of the concept, then the negative instances of the concept over X are the elements of X_2.

15. Positive instances: Given a concept over a class X with partition X_1, X_2 and given that the elements of X_1 are examples of the concept, then the positive instances of the concept are the elements of X_1.

16. Postfeedback interval: The postfeedback interval is the length of time between the presentation of the feedback and the occurrence of the next stimulus.

17. Rational numbers: The set of rational numbers Q is the set of all quotients of the form a/b where a and b are integers and $b \neq 0$.

18. Rules: Conceptual rules are statements which specify how the relevant attributes are combined for use in classifying an instance. Simple rules are typically characterized as conjunctive, disjunctive, conditional, or bioconditional depending on the form of the defining statement.

19. Stimulus interval: The stimulus interval is the length of time the stimulus is available to the subject for inspection.

20. Terminal: The word terminal will be used to refer to the IBM 2741 computer terminal.

21. Universal class: The universal class is the class over which a concept is defined.

A.3 INTRODUCTION AND PRETEST

INTRODUCTION:

Verbal--"This is a computer terminal. It is a special kind of typewriter which has been designed to allow you to communicate with a computer at Ohio State University.

The computer sends electronic signals over the telephone line, the terminal translates the signals to words and types them out.

Whenever you want to send a message to the computer, you simply type your message and the terminal sends it to the computer.

You have been asked to come here to help us find out how effectively a computer can be used to teach things to students. The computer will ask you some mathematics questions and wait for you to answer. Try your best to figure out the answer. If your are not sure of an answer don't hesitate to type in the answer which you think may be correct. The computer will give you hints if you are wrong.

Your work on the computer has noting to do with any of your courses and you are not being graded. Do you understand?

OK! Press the RETURN key and you can being."

From Computer

Welcome to computer-assisted-instruction. This message is being transmitted to you from the Ohio State University Computer Center.

Whenever you are asked to answer a question, simply type your answer and then hit the RETURN key.

OK! Let's see if you understand these directions. What is your first name?
frank

Very good, Frank.
You have successfully communicated with a computer. Remember that after you finish typing an answer you must depress the RETURN key.

The purpose of the time you will spend communicating with the computer is to teach you some mathematics. It is important for you to be careful and do as well as you can. This will help your school to determine the usefulness of computers for teaching mathematics.

Before we can begin, we need to discuss the meaning of the symbols and notation which will be used.

You should be familar with the following symbols:

SYMBOL	MEANING
+	ADD
-	SUBTRACT
*	MULTIPLY

Notice that the symbol, *, means multiply.

Here are some examples:

1. 5 + 7 = 12.

2. 23 - 8 = 15.

3. 9*6 = 54.

Hit the RETURN4key and we will go on.

In the unit you will be studying we will be using a new symbol in addition to the symbols : +, -, and *.

This new symbol is o. It will be used to define several different ways of combining numbers. Here is an example.
The letters a and b may stand for any two numbers.

a o b = 2*a + b + 3.

1 o 6 = ?

The answer to this question is

11

Here is how the answer was figured out:

Since a o b = 2*a + b + 3,
1 o 6 = 2*1 + 6 + 3 and
2*1 + 6 + 3 = 11.

So, 1 o 6 = 11.

Figure out the answers to the following questions. After you have the final answer, type only that answer. Don't show your work, just type the answer.

4 o 2 = ?

6

No, a o b ≠ a + b. Remember that a o b = 2*a + b + 3. In this case a = 4 and b = 2. Now try again.

8

No, a o b ≠ a*b. Recall that a o b = 2*a + b + 3. In this problem a = 4 and b = 2. Try again.

7

Let's look at this problem more carefully. You don't have the right idea. In this case, a = 4 and b = 2. Notice that a o b = 2*a + b + 3, so 4 o 2 = 2*4 + 2 + 3. Now you should be able to do it correctly.

13

Very good. You have the right idea.

Remember that a o b = 2*a + b + 3.

3 o (1 o 5) = ?

30

No! You multiplied 3 and 10. You did the part inside the parentheses correctly. Now you must find the value of 3 o 10. What is 3 o 10 ?

69

The parentheses may be causing some confusion. You must find the value of the part inside the parentheses first, then do the other part. Try again.

19

Excellent!!!

Let's define a o b differently. Suppose a o b = 1 + a + 3*b. Then 2 o 1 = 6. What is 4 o 2 ?

11

Right!!

If a o b = 1 + a + 3*b, (1 o 2) o 1 = ?

12

Very good.

Remember that a o b can be defined in many different ways. Keep this in mind in the next section.

We are coming to a very important part of your session with the computer for today. You will be asked to answer several problems in order to see how wel you understand what has been discussed so far.

The computer will tell you how a o b is defined and 2 examples will be given. You will then be asked to give answers to 2 specific problems. Regardless of your answer the computer will continue on to the next problem.

Please answer each question carefully so that we will be able to determine how well you understand this material.

When you are ready to start hit the RETURN key.

PRETESTS (PCA AND PCB)

Sample Student Interaction

a o b = b, 4 o 7 = 7, 8 o 3 = 3.

1. 5 o 6 = ?

6

2. 2 o (1 o 5) = ?

Sample Programming

```
pretes
  1-  0 pr
  2-  0 cm /pre/a o b = b,  4 o 7 = 7,  8 o 3 = 3.

  2-  1 cc /  1. 5 o 6 = ?

/6
  2-  2 cc /  2. 2 o (1 o 5) = ?

/5
&  3-  0 qu a o b = b,  4 o 7 = 7,  8 o 3 = 3.

&  3-  1  1. 5 o 6 = ?

&  3-  2 ca 6
&  3-  3 ld c0/c3
&  3-  4 ad 1/c1
&  3-  5 ty

&  3-  6 un
```

```
&    3-  7 ld c0/c3
&    3-  8 ad 1/c2
&    3-  9 br pr
&    4-  0 qu
```

2. 2 o (1 o 5) = ?

```
&    4-  1 ca 5
&    4-  2 ld c0/c3
&    4-  3 ad 1/c1
&    4-  4 ty


&    4-  5 un


&    4-  6 ld c0/c3
&    4-  7 ad 1/c2
&    4-  8 br pr
```

FPCA-Pretest Items	PCA Items	PCB Items
$a \circ b = b$, $4 \circ 7 = 7$, $8 \circ 3 = 3$.		
1. $5 \circ 6 = ?$	6	
2. $2 \circ (1 \circ 5) = ?$		5
$a \circ b = 2*(a - b)$, $3 \circ 1 = 4$, $7 \circ 2 = 10$.		
3. $5 \circ 2 = ?$	6	
4. $9 \circ (7 \circ 3) = ?$		2
$a \circ b = 4*a + 2*b$, $5 \circ 3 = 26$, $4 \circ 6 = 28$.		
5. $(2 \circ 1) \circ 3 = ?$		46
6. $3 \circ 6 = ?$	24	
$a \circ b = a*b + 2$, $4 \circ 5 = 22$, $6 \circ 3 = 20$.		
7. $3 \circ 2 = ?$	8	
8. $1 \circ (2 \circ 3) = ?$		10
$a \circ b = 9$, $5 \circ 1 = 9$, $3 \circ 8 = 9$.		
9. $7 \circ 8 = ?$	9	
10. $4 \circ (1 \circ 6) = ?$		9
$a \circ b = 3*(a + b)$, $2 \circ 5 = 21$, $3 \circ 1 = 12$.		
11. $5 \circ 4 = ?$	27	
12. $3 \circ (2 \circ 8) = ?$		99
$a \circ b = 2^a$, $3 \circ 5 = 8$, $2 \circ 7 = 4$.		
13. $3 \circ (2 \circ 7) = ?$		8
14. $1 \circ 7 = ?$	2	
$a \circ b = (a*b)^2$, $2 \circ 3 = 36$, $5 \circ 1 = 25$.		
15. $1 \circ (3 \circ 1) = ?$		81
16. $4 \circ 2 = ?$	64	
$a \circ b$ = a divided by b, $15 \circ 3 = 5$, $24 \circ 6 = 4$.		
17. $14 \circ 2 = ?$	7	
18. $(20 \circ 2) \circ 5 = ?$		2
$a \circ b = a$, $7 \circ 6 = 7$, $4 \circ 9 = 4$.		
19. $(5 \circ 8) \circ 4 = ?$		5
20. $9 \circ 2 = ?$	9	

A.4 TREATMENTS

INTRODUCTION:

Good morning, Frank
I am glad that you could come back.

Today we will be looking at several different ways of defining the operation o. Once again you will be given a definition of a o b followed by 2 examples. Your task is to answer one question for each definition of a o b. The question will require a simple 'yes' or 'no' answer. In fact you won't even have to type the whole word. If your answer is yes, type y and if your answer is no, type n. After you type y or n you will be told if your answer is CORRECT or INCORRECT. The computer will then wait until you depress the RETURN key before continuing on to the next problem. It is important that you remember to hit the RETURN key when you are ready to go on to the next problem. The computer will wait for you to signal it to go on so that you can study the problem you have just answered. In this way you will be given a chance to decide why your answer was correct or incorrect.
Let's do two sample problems.

SAMPLE 1. a o b = a + b, 4 o 5 = 9, 6 o 2 = 8.

a o b = b o a ?

y

Correct.

SAMPLE 2. a o b = a*b, 2 o 3 = 6, 5 o 7 = 35.

a o (b o c) = (a o b) o c ?

y

Correct.

OK. You answered both problems with no trouble. When you are ready to begin hit the RETURN key.

TREATMENT A^+B^+

Sample Student Interaction

1. a o b = 2 + b + a 3 o 4 = 9, 0 o 1 = 3.

a o b = b o a ?

y

Correct.

2. a o b = 3*a*b + 2, 2 o 1 = 8, 3 o 2 = 20.

a o b = b o a ?

y

Correct.

3. a o b = b + a - 2, 4 o 3 = 5, 5 o 1 = 4.

(a o b) o c = a o (b o c) ?

y

Correct.

TREATMENT A^+B^+ (continued)

Sample Programming

```
treat1
  1-  0 cm /repeat/1.   a o b = 2 + b + a,        3 o 4 = 9,        0 o 1 = 3.
  1-  1 cc /


/    a o b = b o a ?/

/y/n
&   2-  0 qu 1.   a o b = 2 + b + a,        3 o 4 = 9,        0 o 1 = 3.
&   2-  1



&   2-  2        a o b = b o a ?
&   2-  3


&   2-  4 ca y
&   2-  5 ld c0/c3
&   2-  6 ad 1/c1
&   2-  7 ty

     Correct.


&   2-  8 ca n
&   2-  9 ld c0/c4
&   2- 10 ad 1/c2
&   2- 11 ty

     Incorrect.


&   2- 12 un Please type either 'y' or 'n.'

&   3-  0 rd


&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld 0/c3
&   3-  6 ld 0/c4
    4-  0 cm /repeat/2.   a o b = 3*a*b + 2,        2 o 1 = 8,        3 o 2 = 20.
    4-  1 cc /


/    a o b = b o a ?/

/y/n
&   5-  0 qu 2.   a o b = 3*a*b + 2,        2 o 1 = 8,        3 o 2 = 20.
&   5-  1
```

TREATMENT A^+B^+ (continued)

Sample Programming (continued)

```
&   5-  2          a o b  =  b o a ?
&   5-  3


&   5-  4 ca y
&   5-  5 ld c0/c3
&   5-  6 ad 1/c1
&   5-  7 ty
     Correct.


&   5-  8 ca n
&   5-  9 ld c0/c4
&   5- 10 ad 1/c2
&   5- 11 ty
     Incorrect.


&   5- 12 un Please type either 'y' or 'n.'

&   6-  0 rd


&   6-  1 ep
&   6-  2 ld c0/c5
&   6-  3 ad c3/c6
&   6-  4 ad c4/c7
&   6-  5 ld 0/c3
&   6-  6 ld 0/c4
    7-  0 cm /repeat/3.   a o b  =  b + a - 2,          4 o 3  =  5,          5 o 1  =  4.
    7-  1 cc /


/     (a o b) o c  =  a o (b o c) ?/

/y/n
&   8-  0 qu 3.   a o b  =  b + a - 2,          4 o 3  =  5,          5 o 1  =  4.
&   8-  1


&   8-  2          (a o b) o c  =  a o (b o c) ?
&   8-  3


&   8-  4 ca y
&   8-  5 ld c0/c3
&   8-  6 ad 1/c1
&   8-  7 ty
```

TREATMENT $A^{+}B^{+}$ (continued)

Sample Programming (continued)

```
Correct.


&   8-  8 ca n
&   8-  9 ld c0/c4
&   8- 10 ad 1/c2
&   8- 11 ty

Incorrect.


&   8- 12 un Please type either 'y' or 'n.'

&   9-  0 rd


&   9-  1 ep
&   9-  2 ld c0/c5
&   9-  3 ad c3/c6
&   9-  4 ad c4/c7
&   9-  5 ld 0/c3
&   9-  6 ld 0/c4
```

TREATMENT $A^{\pm}B^{+}$

Sample Student Interaction

```
1.  a o b = b + a + 2,          3 o 1 = 6,          4 o 5 = 11.

    (a o b) o c = a o (b o c) ?

y


    Correct.

2.  a o b = 2*a + 2*b,          1 o 4 = 10,         3 o 2 = 10.

    a o b = b o a ?

y


    Correct.
```

TREATMENT $A^{\pm}B^{+}$ (continued)

Sample Programming

```
treat2
  1-  0 cm /repeat/1.   a o b = b + a + 2,          3 o 1 = 6,          4 o 5 = 11.
  1-  1 cc /


/     (a o b) o c = a o (b o c) ?/


/y/n
&   2-  0 qu 1.   a o b = b + a + 2,          3 o 1 = 6,          4 o 5 = 11.
&   2-  1



&   2-  2          (a o b) o c = a o (b o c) ?
&   2-  3



&   2-  4 ca y
&   2-  5 ld c0/c3
&   2-  6 ad 1/c1
&   2-  7 ty

     Correct.


&   2-  8 ca n
&   2-  9 ld c0/c4
&   2- 10 ad 1/c2
&   2- 11 xy

     Incorrect.


&   2- 12 un Plejse type either 'y' or 'n.'

&   3-  0 rd



&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld 0/c3
&   3-  6 ld 0/c4
    4-  0 cm /repeat/2.   a o b = 2*a + 2*b,          1 o 4 = 10,          3 o 2 = 10.
    4-  1 cc /


/     a o b = b o a ?/
```

TREATMENT $A^{\pm}B^{+}$ (continued)

Sample Programming (continued)

```
/y/n
&   5-  0 qu 2.   a o b = 2*a + 2*b,          1 o 4 = 10,          3 o 2 = 10.
&   5-  1


&   5-  2          a o b = b o a ?
&   5-  3


&   5-  4 ca y
&   5-  5 ld c0/c3
&   5-  6 ad 1/c1
&   5-  7 ty
```

Correct.

```
&   5-  8 ca n
&   5-  9 ld c0/c4
&   5- 10 ad 1/c2
&   5- 11 ty
```

Incorrect.

```
&   5- 12 un Please type either 'y' or 'n.'
&   6-  0 rd


&   6-  1 ep
&   6-  2 ld c0/c5
&   6-  3 ad c3/c6
&   6-  4 ad c4/c7
      &   6-  5 ld 0/c3
&   6-  6 ld 0/c4
```

TREATMENT $A^{+}B^{\pm}$

Sample Student Interaction

```
1.  a o b = 4*a*b,          2 o 5 = 40,          3 o 1 = 12.


    (a o b) o c = a o (b o c) ?

y


    Correct.
```

TREATMENT $A^{+}B^{\pm}$ (continued)

Sample Student Interaction (continued)

```
2.   a o b = a + b + 1,          6 o 9 = 16,          7 o 3 = 11.

     a o b = b o a ?

y

     Correct.
```

Sample Programming

```
   treat3
     1-  0 cm /repeat/1.   a o b = 4*a*b,          2 o 5 = 40,          3 o 1 = 12.
     1-  1 cc /

/      (a o b) o c = a o (b o c) ?/

/y/n
&    2-  0 qu 1.   a o b = 4*a*b,          2 o 5 = 40,          3 o 1 = 12.
&    2-  1


&    2-  2          (a o b) o c = a o (b o c) ?
&    2-  3


&    2-  4 ca y
&    2-  5 ld c0/c3
&    2-  6 ad 1/c1
&    2-  7 ty
      Correct.


&    2-  8 ca n
&    2-  9 ld c0/c4
&    2- 10 ad 1/c2
&    2- 11 ty
      Incorrect.

 &   2- 12 un Please type either 'y' or 'n.'

 &   3-  0 rd
```

TREATMENT $A^{+}B^{\pm}$ (continued)

Sample Programming (continued)

```
&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld 0/c3
&   3-  6 ld 0/c4
    4-  0 cm /repeat/2.   a o b  =  a + b + 1,        6 o 9  =  16,        7 o 3  =  11.
    4-  1 cc /


/     a o b  =  b o a ?/

/y/n
&   5-  0 qu 2.   a o b  =  a + b  + 1,        6 o 9  =  16,        7 o 3  =  11.
&   5-  1



&   5-  2          a o b  =  b o a ?
&   5-  3


&   5-  4 ca y
&   5-  5 ld c0/c3
&   5-  6 ad 1/c1
&2  5-  7 ty
```

Correct.

```
&   5-  8 ca n
&   5-  9 ld c0/c4
&   5- 10 ad 1/c2
&   5- 11 ty
```

Incorrect.

```
&   5- 12 un Please type either 'y' or  'n.'

&   6-  0 rd


&   6-  1 ep
&   6-  2 ld c0/c5
&   6-  3 ad c3/c6
&   6-  4 ad c4/c7
&   6-  5 ld 0/c3
&   6-  6 ld 0/c4
```

TREATMENT $A^{+}B^{+}$

Sample Student Interaction

```
1.  a o b = 4*(a + b),          3 o 5 = 32,          4 o 2 = 24.

    (a o b) o c = a o (b o c) ?

n

    Correct.


2.  a o b = 2*a + 2*b,          4 o 8 = 24,          1 o 9 = 20.

    a o b = b o a ?

y

    Correct.
```

Sample Programming

```
    1-  0 cm /assoc/1.   a o b = 4*(a + b),          3 o 5 = 32,          4 o 2 = 24.
    1-  1 cc /n/y
&   2-  0 qu 1.   a o b = 4*(a + b),          3 o 5 = 32,          4 o 2 = 24.
&   2-  1

&   2-  2          (a o b) o c = a o (b o c) ?
&   2-  3

&   2-  4 ca n
&   2-  5 ld c0/c3
&   2-  6 ad 1/c1
&   2-  7 ty

&   2-  8          Correct.
```

TREATMENT $A^{\pm}B^{\pm}$ (continued)

Sample Programming (continued)

```
&   2-  9 ca y
&   2- 10 ld c0/c4
&   2- 11 ad 1/c2
&   2- 12 ty

&   2- 13          Incorrect.

&   2- 14 un Please type either 'y' or 'n.'

&   3-  0 rd

&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld 0/c3
&   3-  6 ld 0/c4
    4-  0 cm /comm/2.   a o b  =  2*a + 2*b,          4 o 8  =  24,          1 o 9  =  20.
    4-  1 cc /y/n
&   5-  0 qu 2.   a o b  =  2*a + 2*b,          4 o 8  =  24,          1 o 9  =  20.
&   5-  1

&   5-  2          a o b  =  b o a ?

&   5-  3

&   5-  4 ca y
&   5-  5 ld c0/c3
&   5-  6 ad 1/c1
&   5-  7 ty

&   5-  8          Correct.

&   5-  9 ca n
&   5- 10 ld c0/c4
&   5- 11 ad 1/c2
&   5- 12 ty

&   5- 13          Incorrect.

&   5- 14 un Please type either 'y' or 'n.'

&   6-  0 rd

&   6-  1 ep
&   6-  2 ld c0/c5
&   6-  3 ad c3/c6
&   6-  4 ad c4/c7
&   6-  5 ld 0/c3
&   6-  6 ld 0/c4
```

INSTANCES BY TREATMENTS

Treatment A^+B^+

			Concept A	Concept B
SAMPLE 1.	a o b = a + b,	4 o 5 = 9, 6 o 2 = 8.		
	a o b = b o a ?			
SAMPLE 2.	a o b = a*b,	2 o 3 = 6, 5 o 7 = 35.		
	a o (b o c) = (a o c) o c ?			
1.	a o b = 2 + b + a,	3 o 4 = 9, 0 o 1 = 3.		
	a o b = b o a ?		+	
2.	a o b = 3*a*b + 2,	2 o 1 = 8, 3 o 2 = 20.		
	a o b = b o a ?		+	
3.	a o b = b + a - 2,	4 o 3 = 5, 5 o 1 = 4.		
	(a o b) o c = a o (b o c) ?			+
4.	a o b = the smaller of a and b,	5 o 4 = 4, 3 o 3 = 3.		
	(a o b) o c = a o (b o c)?			+
5.	a o b = 1 + a + b,	4 o 3 = 8, 2 o 6 = 9.		
	(a o b) o c = a o (b o c) ?			+
6.	a o b = 3 + 2*a*b,	2 o 2 = 11, 3 o 4 = 27.		
	a o b = b o a ?		+	
7.	a o b = 4*b*a,	1 o 5 = 20, 3 o 2 = 24.		
	(a o b) o c = a o (b o c) ?			+
8.	a o b = 2*(a + b) + 1,	4 o 6 = 21, 5 o 3 = 17.		
	a o b = b o a?		+	
9.	a o b = the larger of a and b,	6 o 8 = 8, 7 o 4 = 7.		
	(a o b) o c = a o (b o c) ?			+

Treatment A^+B^+ (continued)

				Concept A	Concept B
10.	a o b = (a + b) - 3,	3 o 4 = 4,	1 o 2 = 0.		
	a o b = b o a ?			+	
11.	a o b = a*b + 4,	5 o 2 = 14,	3 o 4 = 16.		
	a o b = b o a ?			+	
12.	a o b = 0,	7 o 6 = 0,	2 o 9 = 0.		
	a o b = b o a ?			+	
13.	a o b = a + b + 5,	2 o 6 = 13,	5 o 1 = 11.		
	(a o b) o c = a o (b o c) ?				+
14.	a o b = 1,	4 o 3 = 1,	2 o 7 = 1.		
	(a o b) o c = a o (b o c) ?				+
15.	a o b = 3*b*a,	2 o 3 = 18,	4 o 1 = 12.		
	(a o b) o c = a o (b o c) ?				+
16.	a o b = 2,	4 o 3 = 2,	5 o 7 = 2.		
	a o b = b o a ?			+	
17.	a o b = a + b - 2,	5 o 3 = 6,	2 o 7 = 7.		
	(a o b) o c = a o (b o c) ?				+
18.	a o b = a*b - 1,	4 o 5 = 19,	8 o 3 = 23.		
	a o b = b o a ?			+	
19.	a o b = a + b + 3,	1 o 9 = 13,	7 o 0 = 10.		
	(a o b) o c = a o (b o c) ?				+
20.	a o b = 4*a*b,	2 o 1 = 8,	3 o 3 = 36		
	a o b = b o a ?			+	
21.	a o b = 5*(a + b),	4 o 5 = 45,	3 o 1 = 20.		
	a o b = b o a ?			+	

Treatment A^+B^+ (continued)

		Concept A	Concept B
22.	a o b = b, 5 o 2 = 5, 7 o 9 = 7. (a o b) o c = a o (b o c) ?		+
23.	a o b = 2*a + 2*b, 3 o 5 = 16, 0 o 5 = 10. a o b = b o a ?	+	
24.	a o b = 5, 2 o 9 = 5, 9 o 6 = 5. (a o b) o c = a o (b o c) ?		+
25.	a o b = b*a, 3 o 6 = 18, 4 o 2 = 8. (a o b) o c = a o (b o c) ?		+
26.	a o b = b, 4 o 7 = 7, 3 o 2 = 2. (a o b) o c = a o (b o c) ?		+
27.	a o b = 3, 1 o 8 = 3, 7 o 5 = 3. a o b = b o a ?	+	
28.	a o b = 6*a*b, 3 o 1 = 18, 2 o 5 = 60. (a o b) o c = a o (b o c) ?		+
29.	a o b = a, if a ≠ b, otherwise a o b = b, 4 o 3 = 4, 6 o 6 = 6. (a o b) o c = a o (b o c) ?		+
30.	a o b = 2*(a + b), 3 o 5 = 16, 4 o 1 = 10. a o b = b o a ?	+	
31.	a o b = 4*a*b - 3, 2 o 5 = 37, 4 o 1 = 13. a o b = b o a ?	+	
32.	a o b = 2, 7 o 8 = 2, 4 o 3 = 2. (a o b) o c = a o (b o c) ?		+

Treatment A^+B^+ (continued)

		Concept A	Concept B
33.	$a \circ b = 2*(a + b + 1)$, $3 \circ 5 = 18$, $6 \circ 1 = 16$. $a \circ b = b \circ a$?	+	
34.	$a \circ b = b + a + 1$, $1 \circ 0 = 2$, $4 \circ 7 = 12$. $(a \circ b) \circ c = a \circ (b \circ c)$?		+
35.	$a \circ b = 7 + a + b$, $0 \circ 5 = 12$, $8 \circ 3 = 18$. $a \circ b = b \circ a$?	+	
36.	$a \circ b = 5*b*a$, $5 \circ 1 = 25$, $2 \circ 6 = 60$. $(a \circ b) \circ c = a \circ (b \circ c)$?		+
37.	$a \circ b = b$, if $b \neq a$, otherwise $a \circ b = a$, $5 \circ 5 = 5$, $7 \circ 2 = 2$. $(a \circ b) \circ c = a \circ (b \circ c)$?		+
38.	$a \circ b = 4 + 3*a + 3*b$, $0 \circ 4 = 16$, $2 \circ 3 = 19$. $a \circ b = b \circ a$?	+	
39.	$a \circ b = a + b + 3$, $4 \circ 6 = 13$, $7 \circ 9 = 19$. $a \circ b = b \circ a$?	+	
40.	$a \circ b = 6$, $5 \circ 8 = 6$, $7 \circ 4 = 6$. $a \circ b = b \circ a$?	+	

Treatment $A^{\pm}B^{+}$

				Concept A	Concept B
SAMPLE 1.	a o b a + b,	4 o 5 = 9,	6 o 2 = 8.		
	a o b = b o a ?				
SAMPLE 2.	a o b a*b,	2 o 3 = 6,	5 o 7 = 35.		
	a o (b o c) = (a o b) o c ?				
1.	a o b = b + a + 2,	3 o 1 = 6,	4 o 5 = 11.		
	(a o b) o c = a o (b o c) ?				+
2.	a o b = 2*a + 2*b,	1 o 4 = 10,	3 o 2 = 10.		
	a o b = b o a ?			+	
3.	a o b = 4*a*b,	3 o 2 = 24,	4 o 5 = 80.		
	(a o b) o c = a o (b o c) ?				+
4.	a o b = 2*a + 3*b,	2 o 3 = 13,	4 o 1 = 11.		
	a o b = b o a ?			-	
5.	a o b = a divided by b,	21 o 3 = 7,	10 o 4 = 2.5.		
	a o b = b o a ?			-	
6.	a o b = a*b - 1,	2 o 4 = 7,	5 o 3 = 14.		
	a o b = b o a ?			+	
7.	a o b = a + b + 1,	6 o 9 = 16,	7 o 4 = 12.		
	(a o b) o c = a o (b o c) ?				+
8.	a o b = (a - b) + 1,	5 o 2 = 4,	3 o 3 = 1.		
	a o b - b o a ?			-	
9.	a o b = 4 + a + b,	1 o 5 = 10,	3 o 2 = 9.		
	(a o b) o c = a o (b o c) ?				+
10.	a o b = 3*a*b,	4 o 2 = 24,	1 o 7 = 21.		
	a o b = b o a ?			+	

Treatment $A^{\pm}B^{+}$ (continued)

		Concept A	Concept B
11.	a o b = 1, 6 o 9 = 1, 7 o 3 = 1.		
	(a o b) o c = a o (b o c) ?		+
12.	a o b = 5 + (a + b), 2 o 5 = 12, 7 o 3 = 15.		
	a o b = b o a ?	+	
13.	a o b = a if a ≠ b, otherwise a o b = b, 4 o 9 = 4, 5 o 5 = 5.		
	(a o b) o c = a o (b o c) ?		+
14.	a o b = a*b + 1, 3 o 4 = 13, 7 o 2 = 15.		
	(a o b) o c = a o (b o c) ?		-
15.	a o b = a + b - 1, 4 o 8 = 11, 0 o 1 = 0.		
	a o b = b o a ?	+	
16.	a o b = 6*b*a, 2 o 3 = 36, 4 o 1 = 24.		
	(a o b) o c = a o (b o c) ?		+
17.	a o b = 4*b + a, 3 o 6 = 27, 5 o 2 = 13.		
	a o b = b o a ?	-	
18.	a o b = 2, 9 o 4 = 2, 5 o 3 = 2.		
	(a o b) o c = a o (b o c) ?		+
19.	a o b = a, 8 o 3 = 8, 1 o 9 = 1.		
	a o b = b o a ?	-	
20.	a o b = a + b + 3, 4 o 2 = 9, 8 o 9 = 20.		
	(a o b) o c = a o (b o c) ?		+
21.	a o b = 5*(a + b), 2 o 7 = 45, 5 o 3 = 40.		
	a o b = b o a ?	+	
22.	a o b = a*b + 4, 3 o 7 = 25, 5 o 2 = 14.		
	a o b = b o a ?	+	

Treatment $A^{\pm}B^{+}$ (continued)

		Concept A	Concept B
23.	a o b = 3*a*b, 1 o 7 = 21, 9 o 2 = 54.		
	(a o b) o c = a o (b o c) ?		+
24.	a o b = a, 5 o 6 = 5, 9 o 3 = 9.		
	(a o b) o c = a o (b o c) ?		+
25.	a o b = b if b ≠ a, otherwise a o b = a, 5 o 9 = 9, 2 o 2 = 2.		
	a o b = b o a ?		-
26.	a o b = a - b, 7 o 3 = 4, 9 o 8 = 1.		
	a o b = b o a ?		-
27.	a o b = a if a ≥ b, otherwise a o b = b, 2 o 4 = 4, 5 o 3 = 5.		
	(a o b) o c = a o (b o c) ?		+
28.	a o b = a + 3*b, 1 o 4 = 13, 3 o 2 = 9.		
	a o b = b o a ?		-
29.	a o b = 6, 5 o 9 = 6, 3 o 0 = 6.		
	a o b = b o a ?		+
30.	a o b = 2*a*b, 2 o 7 = 28, 5 o 4 = 40.		
	(a o b) o c = a o (b o c) ?		+
31.	a o b = b if b ≠ a, otherwise, a o b = a, 3 o 3 = 3, 4 o 5 = 5.		
	(a o b) o c = a o (b o c) ?		+
32.	a o b = 0, 5 o 8 = 0, 3 o 3 = 0.		
	a o b = b o a ?		+
33.	a o b = 5, 4 o 9 = 5, 3 o 1 = 5.		
	(a o b) o c = a o (b o c) ?		+

Treatment $A^{+}B^{+}$ (continued)

		Concept A	Concept B
34.	$a \circ b = 7 + a + b$, $2 \circ 6 = 15$, $8 \circ 3 = 18$.		
	$a \circ b = b \circ a$?	+	
35.	$a \circ b = a$ if $a < b$, otherwise $a \circ b = b$, $4 \circ 2 = 2$, $3 \circ 8 = 3$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
36.	$a \circ b = b$, $7 \circ 9 = 9$, $8 \circ 3 = 3$.		
	$a \circ b = b \circ a$?	-	
37.	$a \circ b = a + b + 8$, $4 \circ 7 = 19$, $6 \circ 3 = 17$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
38.	$a \circ b = 0$, $9 \circ 5 = 0$, $1 \circ 6 = 0$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
39.	$a \circ b = b$, $4 \circ 7 = 7$, $8 \circ 3 = 3$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
40.	$a \circ b = a$ if $a \neq b$, otherwise, $a \circ b = b$, $2 \circ 8 = 2$, $4 \circ 1 = 4$.		
	$a \circ b = b \circ a$?	-	

Treatment $A^{+}B^{\pm}$

			Concept A	Concept B
SAMPLE 1.	a o b a + b,	4 o 5 = 9, 6 o 2 = 8.		
	a o b = b o a ?			
SAMPLE 2.	a o b = a*b,	2 o 3 = 6, 5 o 7 = 35.		
	a o (b o c) = (a o b) o c ?			
1.	a o b = 4*a*b,	2 o 5 = 40, 3 o 1 = 12.		
	(a o b) o c = a o (b o c) ?			+
2.	a o b = a + b + 1,	6 o 9 = 16, 7 o 3 = 11.		
	a o b = b o a ?		+	
3.	a o b = a*b + 4,	3 o 9 = 31, 8 o 2 = 20.		
	a o b = b o a ?		+	
4.	a o b = a,	7 o 4 = 7, 10 o 8 = 10.		
	(a o b) o c = a o (b o c) ?			+
5.	a o b = 6,	5 o 9 = 6, 5 o 3 = 6.		
	a o b = b o a ?		+	
6.	a o b = 0,	4 o 9 = 0, 5 o 3 = 0.		
	a o b = b o a ?		+	
7.	a o b = a - b,	6 o 4 = 2, 13 o 6 = 7.		
	(a o b) o c = a o (b o c) ?			-
8.	a o b = a*b - 1,	4 o 7 = 27, 6 o 2 = 11.		
	a o b = b o a ?		+	
9.	a o b = 2*a*b - 1,	4 o 8 = 63, 5 o 2 - 19.		
	(a o b) o c = a o (b o c) ?			-
10.	a o b = a*b + 3,	2 o 9 = 21, 4 o 1 = 7.		
	(a o b) o c = a o (b o c) ?			-

Treatment $A^{+}B^{-}$ (continued)

		Concept A	Concept B
11.	a o b = 2*a + 2*b, 3 o 7 = 20, 5 o 2 = 14. a o b = b o a ?	+	
12.	a o b = 4*(a + b), 4 o 2 = 24, 5 o 6 = 44. a o b = b o a ?	+	
13.	a o b = 3*a*b + 2, 4 o 1 = 14, 7 o 3 = 65. a o b = b o a ?	+	
14.	a o b = 6*(a + b), 3 o 2 = 30, 5 o 4 = 54. a o b = b o a ?	+	
15.	a o b = a if a ≠ b, otherwise a o b = b, 3 o 5 = 3, 4 o 4 = 4. (a o b) o c = a o (b o c) ?	+	
16.	a o b = 5*(a + b) - 2, 4 o 9 = 63, 6 o 4 = 48. (a o b) o c = a o (b o c) ?	-	
17.	a o b = 2, 3 o 7 = 2, 9 o 2 = 2. (a o b) o c = a o (b o c) ?	+	
18.	a o b = 3, 6 o 8 = 3, 7 o 9 = 3. a o b = b o a ?	+	
19.	a o b = 3*b + 2*a, 3 o 7 = 27, 5 o 6 = 28. (a o b) o c = a o (b o c) ?	-	
20.	a o b = b if b ≠ a, otherwise a o b = a, 4 o 8 = 8, 5 o 5 = 5. (a o b) o c = a o (b o c) ?	+	
21.	a o b = 1, 2 o 9 = 1, 4 o 5 = 1. a o b = b o a ?	+	

Treatment $A^{+}B^{\pm}$ (continued)

		Concept A	Concept B
22.	a o b = 5*a + 5*b, 3 o 5 = 40, 2 o 1 = 15.		
	a o b = b o a ?	+	
23.	a o b = a divided by b, 7 o 3 = 2, 18 o 6 = 3.		
	(a o b) o c = a o (b o c) ?		-
24.	a o b = 3*a*b, 2 o 8 = 48, 6 o 5 = 90.		
	a o b = b o a ?	+	
25.	a o b = a + b - 1, 4 o 7 = 10, 5 o 2 = 6,		
	(a o b) o c = a o (b o c) ?		+
26.	a o b = 2*a*b, 4 o 3 = 24, 5 o 6 = 60.		
	(a o b) o c = a o (b o c) ?		+
27.	a o b = 3 + 2*a + 2*b, 1 o 9 = 23, 4 o 2 = 15.		
	a o b = b o a ?	+	
28.	a o b = 2*(a + b) + 1, 3 o 7 = 21, 5 o 2 = 15.		
	(a o b) o c = a o (b o c) ?		-
29.	a o b = a + b + 3, 2 o 8 = 13, 5 o 1 = 9.		
	a o b = b o a ?	+	
30.	a o b = 5, 6 o 9 = 5, 7 o 3 = 5.		
	(a o b) o c = a o (b o c) ?		+
31.	a o b = b - a, 5 o 9 = 4, 3 o 6 = 3.		
	(a o b) o c = a o (b o c) ?		-
32.	a o b = 2*(a + b) + 1, 4 o 2 = 13, 6 o 9 = 31.		
	a o b = b o a ?	+	

Treatment $A^{+}B^{\pm}$ (continued)

			Concept A	Concept B
33.	a o b = 3*b - a,	4 o 3 = 5, 5 o 4 = 7.		
	(a o b) o c = a o (b o c) ?			-
34.	a o b = 3*(a + b + 2),	2 o 5 = 27, 4 o 1 = 21.		
	a o b = b o a ?		+	
35.	a o b = b,	3 o 8 = 8, 7 o 4 = 4.		
	(a o b) o c = a o (b o c) ?			+
36.	a o b = 2*a + 5*b,	4 o 3 = 23, 6 o 2 = 22.		
	(a o b) o c = a o (b o c) ?			-
37.	a o b = 4*a*b - 1,	2 o 3 = 23, 5 o 1 = 19.		
	a o b = b o a ?		+	
38.	a o b = 0,	5 o 9 = 0, 4 o 2 = 0.		
	(a o b) o c = a o (b o c) ?			+
39.	a o b = (a + b) - 2,	4 o 5 = 7, 2 o 8 = 8.		
	a o b = b o a ?		+	
40.	a o b = 3 + 2*a*b,	4 o 1 = 11, 5 o 8 = 83.		
	a o b = b o a ?		+	

Treatment $A^{\pm}B^{\pm}$

			Concept A	Concept B
SAMPLE 1.	a o b = a + b,	4 o 5 = 9, 6 o 2 = 8.		
	a o b = b o a ?			
SAMPLE 2.	a o b = a*b,	2 o 3 = 6, 5 o 7 = 35.		
	a o (b o c) = (a o b) o c ?			
1.	a o b = 4*(a + b),	3 o 5 = 32, 4 o 2 = 24.		
	(a o b) o c = a o (b o c) ?			-
2.	a o b = 2*a + 2*b,	4 o 8 = 24, 1 o 9 = 20.		
	a o b = b o a ?		+	
3.	a o b = 4*a*b,	5 o 2 = 40, 3 o 1 = 12.		
	(a o b) o c = a o (b o c) ?			+
4.	a o b = 2*a + 3*b,	5 o 3 = 19, 7 o 2 = 20.		
	a o b = b o a ?		-	
5.	a o b = a divided by b,	16 o 4 = 4, 14 o 5 = 2.30.		
	a o b = b o a ?		-	
6.	a o b = a*b - 1,	4 o 7 = 27, 6 o 8 = 47.		
	a o b = b o a ?		+	
7.	a o b = a + b - 1,	7 o 9 = 15, 3 o 8 = 10.		
	(a o b) o c = a o (b o c) ?			+
8.	a o b = 2*(a - b) + 1,	4 o 1 = 7, 8 o 3 = 11.		
	a o b = b o a ?		-	
9.	a o b = 2*a*b - 1,	4 o 7 = 55, 2 o 1 = 3.		
	(a o b) o c = a o (b o c) ?			-
10.	a o b = 3*a*b,	8 o 2 = 48, 2 o 6 = 36.		
	a o b = b o a ?		+	

Treatment $A^{\pm}B^{\pm}$ (continued)

		Concept A	Concept B
11.	$a \circ b = 5*(a + b) - 2$, $3 \circ 5 = 38$, $1 \circ 4 = 23$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
12.	$a \circ b = 3 + 2*(a + b)$, $3 \circ 8 = 25$, $7 \circ 2 = 21$.		
	$a \circ b = b \circ a$?	+	
13.	$a \circ b = a$ if $a \neq b$, otherwise $a \circ b = b$, $3 \circ 3 = 3$, $4 \circ 7 = 4$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
14.	$a \circ b = a - b$, $12 \circ 5 = 7$, $17 \circ 4 = 13$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
15.	$a \circ b = 3*b - 2*a$ $5 \circ 8 = 14$, $7 \circ 6 = 4$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
16.	$a \circ b = a + b + 1$, $9 \circ 8 = 18$, $4 \circ 2 = 7$.		
	$a \circ b = b \circ a$?	+	
17.	$a \circ b = 4*b + a$, $2 \circ 8 = 34$, $7 \circ 3 = 19$.		
	$a \circ b = b \circ a$?	-	
18.	$a \circ b = 2$, $7 \circ 9 = 2$, $4 \circ 8 = 2$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
19.	$a \circ b = a$, $9 \circ 2 = 9$, $4 \circ 8 = 4$.		
	$a \circ b = b \circ a$?	-	
20.	$a \circ b = 2*b - a$, $5 \circ 4 = 3$, $9 \circ 7 = 5$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
21.	$a \circ b = 5*(a + b)$, $4 \circ 8 = 60$, $7 \circ 3 = 50$.		
	$a \circ b = b \circ a$?	+	

Treatment $A^{\pm}B^{\pm}$ (continued)

		Concept A	Concept B
22.	a o b = a*b + 4, 7 o 9 = 67, 8 o 1 = 12.		
	a o b = b o a ?	+	
23.	a o b = a*b + 3, 5 o 7 = 38, 9 o 2 - 21.		
	(a o b) o c = a o (b o c) ?		-
24.	a o b = b, 8 o 3 = 3, 5 o 2 = 2.		
	a o b = b o a ?	-	
25.	a o b = b, if b ≠ a, otherwise a o b = a, 7 o 7 = 7, 4 o 9 = 9.		
	a o b = b o a ?	-	
26.	a o b = a - b, 8 o 2 = 6, 9 o 8 = 1.		
	a o b = b o a ?	-	
27.	a o b = a divided by b, 9 o 3 = 3, 12 o 8 = 1.5.		
	(a o b) o c = a o (b o c) ?		-
28.	a o b = 3*a + 2*b, 4 o 3 = 18, 6 o 8 = 34.		
	a o b = b o a ?	-	
29.	a o b = 6, 4 o 9 = 6, 8 o 3 = 6.		
	a o b = b o a ?	+	
30.	a o b = 2*a*b, 5 o 6 = 60, 2 o 9 = 36.		
	(a o b) o c = a o (b o c) ?		+
31.	a o b = b, if b ≠ a, otherwise a o b = a, 6 o 8 = 8, 2 o 2 = 2.		
	(a o b) o c = a o (b o c) ?		+
32.	a o b = 0, 8 o 9 = 0, 3 o 7 = 0.		
	a o b = b o a ?	+	

Treatment $A^{\pm}B^{\pm}$ (continued)

		Concept A	Concept B
33.	a o b = 5, 4 o 7 = 5, 6 o 9 = 5. (a o b) o c = a o (b o c) ?		+
34.	a o b = 7 + a + b, 4 o 7 = 18, 9 o 6 = 22. a o b = b o a ?	+	
35.	a o b = 3*b - a 4 o 7 = 17, 6 o 4 = 6. (a o b) o c = a o (b o c) ?		-
36.	a o b = a, 9 o 2 = 9, 4 o 5 = 4. (a o b) o c = a o (b o c) ?		+
37.	a o b = 2*a + 5*b, 4 o 3 = 23, 6 o 2 = 22. (a o b) o c = a o (b o c) ?		-
38.	a o b = 0, 8 o 4 = 0, 5 o 7 = 0. (a o b) o c = a o (b o c) ?		+
39.	a o b = a if a ≠ b, otherwise, a o b = b, 3 o 7 = 3, 5 o 2 = 2. a o b = b o a ?	-	
40.	a o b = b, 7 o 3 = 3, 4 o 5 = 5. (a o b) o c = a o (b o c) ?		+

A.5 POSTTEST (POA AND POB)

INTRODUCTION:

Welcome back, Frank. How have you been since you were here the last time?
(Type a one word description.)

fine

Good.

As you should know, today will be your last session with the computer terminal. The material which will be presented will be very similar to what you have already seen. The operation o will be defined followed by 2 examples. You will then be asked to respond to 2 questions about the way o has been defined. Remember to follow these simple instructions in answering each question:

1. Be very careful and take as much time as you need.
2. You will be asked to give either a 'yes' or 'no' answer to each problem. Simply type 'y' if your answer is 'yes,' and 'n' if your answer is 'no.'
3. After you type your answer be sure to depress the RETURN key.

You will notice that you will not be told whether your answer is CORRECT or INCORRECT. So, there will be no need for you to hit the RETURN key in order to go on to the next item. Hit the RETURN key and we will begin.

POSTTEST

Sample Student Interaction

1. a o b = 2(a - b), 3 o 1 = 4, 5 o 2 = 6.

(a o b) o c = a o (b o c) ?

y

a o b = b o a ?

y

POSTTEST (continued)

Sample Student Interaction (continued)

```
2.  a o b = (a - b) + 3,          7 o 4 = 6,          3 o 1 = 5.

    (a o b) o c = a o (b o c) ?
```

Sample Programming

```
    1-  0 pr
    1-  1 ty 1.  a o b = 2(a - b),          3 o 1 = 4,          5 o 2 = 6.


    2-  0 cm /postma/n/y
&   3-  0 qu      (a o b) o c = a o (b o c) ?


&   3-  1 ca n
&   3-  2 ld c0/c3
&   3-  3 ad 1/c1
&   3-  4 ty

&   3-  0 qu      (a o b) o c = a o (b o c) ?


&   3-  1 ca n
&   3-  2 ld c0/c3
&   3-  3 ad 1/c1
&   3-  4 ty


&   3-  5 ca y
&   3-  6 ld c0/c4
&   3-  7 ad 1/c2
&   3-  8 ty


&   3-  9 un Please type either 'y' or  'n.'

    4-  0 cm /postmc/n/y
&   5-  0 qu      a o b = b o a ?
```

POSTTEST (continued)

Sample Programming (continued)

```
&   5-  1 ca n
&   5-  2 ld c0/c3
&   5-  3 ad 1/c1
&   5-  4 ty


&   5-  5 ca y
&   5-  6 ld c0/c4
&   5-  7 ad 1/c2



&   5-  8 ty

&   5-  9 un Please type either 'y' or 'n.'
```

Posttest (POA and POB)

	Concept A	Concept B
1. $a \circ b = 2(a - b)$, $3 \circ 1 = 4$, $5 \circ 2 = 6$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	-	
2. $a \circ b = (a - b) + 3$, $7 \circ 4 = 6$, $3 \circ 1 = 5$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	-	
3. $a \circ b = b$, $4 \circ 7 = 7$, $8 \circ 3 = 3$.		
$a \circ b = b \circ a$?	-	
$(a \circ b) \circ c = a \circ (b \circ c)$?		+
4. $a \circ b = 3(a + b)$, $2 \circ 5 = 21$, $3 \circ 1 = 12$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	+	
5. $a \circ b$ = a divided by b, $15 \circ 3 = 5$, $24 \circ 6 = 4$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	-	
6. $a \circ b = (a*b)^2$, $2 \circ 3 = 36$, $5 \circ 1 = 25$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	+	
7. $a \circ b = a$, $7 \circ 6 = 7$, $4 \circ 9 = 9$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		+
$a \circ b = b \circ a$?	-	
8. $a \circ b = a*b + 2$, $4 \circ 5 = 22$, $6 \circ 3 = 20$.		
$(a \circ b) \circ c = a \circ (b \circ c)$?		-
$a \circ b = b \circ a$?	+	

Posttest (POA and POB) (Continued)

		Concept A	Concept B
9.	$a \circ b = 2^a$, $3 \circ 5 = 8$, $2 \circ 7 = 4$.		
	$a \circ b = b \circ a$?	-	
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
10.	$a \circ b$ = the smaller of a and b, $4 \circ 4 = 4$, $9 \circ 5 = 5$.		
	$a \circ b = b \circ a$?	+	
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
11.	$a \circ b = (a*b)$ divided by 3, $2 \circ 6 = 4$, $9 \circ 4 = 12$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
	$a \circ b = b \circ a$?	+	
12.	$a \circ b = 9$, $5 \circ 1 = 9$, $3 \circ 8 = 9$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
	$a \circ b = b \circ a$?	+	
13.	$a \circ b = 4*a + 4*b$, $2 \circ 3 = 20$, $5 \circ 1 = 24$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
	$a \circ b = b \circ a$?	+	
14.	$a \circ b = a^2 + b^2$, $3 \circ 2 = 17$, $4 \circ 2 = 24$.		
	$a \circ b = b \circ a$?	-	
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
15.	$a \circ b$ = the larger of a and b, $9 \circ 3 = 9$, $4 \circ 6 = 6$.		
	$a \circ b = b \circ a$?	+	
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+

Posttest (POA and POB) (Continued)

		Concept A	Concept B
16.	$a \circ b = (a - b)^2$, $5 \circ 2 = 9$, $6 \circ 4 = 4$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
	$a \circ b = b \circ a$?	-	
17.	$a \circ b = a^b$, $2 \circ 3 = 8$, $4 \circ 2 = 16$.		
	$a \circ b = b \circ a$?	-	
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
18.	$a \circ b$ = (a + b) divided by 2, $5 \circ 7 = 6$, $8 \circ 6 = 7$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
	$a \circ b = b \circ a$?	+	
19.	$a \circ b = 4*a + 2*b$, $5 \circ 3 = 26$, $4 \circ 6 = 28$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		-
	$a \circ b = b \circ a$?	-	
20.	$a \circ b = 3*a*b$, $4 \circ 5 = 60$, $2 \circ 1 = 6$.		
	$(a \circ b) \circ c = a \circ (b \circ c)$?		+
	$a \circ b = b \circ a$?	+	

A.6 SUBJECTS DROPPED

Subject	Treatment	Reason for Drop
*1101	A+B+	Data accidentally destroyed by computer center personnel (1-19-72).
1104		Randomly dropped to achieve equal cell sizes.
1105		Randomly dropped to achieve equal cell sizes.
1109		Randomly dropped to achieve equal cell sizes.
1110		Randomly dropped to achieve equal cell sizes.
*1201	A±B+	Absent three times for treatment session.
*1207		Dropped out of school.
1209		Randomly dropped to achieve equal cell sizes.
1214		Consistently absent from school.
1218		Consistently absent from school.
*2103	A+B±	Data accidently destroyed by computer center personnel (1-19-72).
*2106		Consistently absent from school.
2107		Data accidently destroyed by computer center personnel (1-19-72).
2111		Consistently absent from school.
2120		Computer system failed.
*2202	A±B±	Data accidently destroyed by computer center personnel (1-19-72).
2206		Terminal errors.
2208		Data accidently destroyed by computer center personnel (1-19-72).
2213		Randomly dropped to achieve equal cell sizes.
*2216		Absent three times for treatment session.
2219		Randomly dropped to achieve equal cell sizes.

* Subject identification number was reassigned to another subject in same treatment.

Summary

Treatment	Number Randomly Dropped	Total Number Dropped
A+B+	4	5
A±B+	1	5
A+B±	0	5
A±B±	2	6
Total	7	21

A.7 DESCRIPTION AND TIME SCHEDULE FOR PILOT STUDIES, DATA COLLECTION, AND ANALYSIS

Dates	Event
9-68 to 5-69	The study conducted by the author at Marshall-University High School during the academic year 1968-1969 is viewed as a pilot for this study (Shumway, 1970 and 1971). The sample was 84 eighth grade mathematics students randomly assigned to four classes. Two teachers each taught two classes, one receiving a treatment of all positive instances and the other a treatment of both positive and negative instances. The duration of the experiment was 65 school days. Among other concepts, the concepts of commutativity, associativity, closure, identity element, inverse element and distributivity were used. A criterion measure was developed for these concepts and many binary operations were invented to provide appropriate positive and negative instances. Negative instances appeared to discourage the common error of overgeneralization. Commutativity and associativity were deemed appropriate concepts for further study.
6-71 to 9-71	Pretest, treatment and posttest material were programmed in Coursewriter III.
9-71 to 11-71	Preliminary pilot and debugging of computerized materials.
11-71 to 12-71	Pilot Study II. Forty-four college students were randomly assigned in equal numbers to the four treatment groups. Extensive comments were solicited. Reliability estimates for the pre- and posttests were calculated. Based on this pilot data, the number of items in each pretest was cut from 20 to 10.
1-72 to 4-72	Data collected for actual study.
5-72 to 6-72	Analysis and final report.

A.8 GLOSSARY OF VARIABLE NAMES

Achievement Variables

Name	Description
PCA	Pretest, Calculations without parentheses
PCB	Pretest, Calculations with parentheses
TA	Treatment, Concept A
TB	Treatment, Concept B
POA	Posttest, Concept A
POB	Posttest, Concept B

Temporal Variables

Name	Description
PCSIA	Stimulus Intervals for PCA
PCSIB	Stimulus Intervals for PCB
TSIA	Stimulus Intervals for TA
TSIB	Stimulus Intervals for TB
TPIA	Postfeedback Intervals for TA
TPIB	Postfeedback Intervals for TB
POSIA	Stimulus Intervals for POA
POSIB	Stimulus Intervals for POB

A.9 CORRELATION MATRIX FOR ALL VARIABLES

TABLE XVII

CORRELATION MATRIX FOR ALL VARIABLES

Variable	PCA	PCB	POB	POA	TB	TA	PCSIA	PCSIB	POSIB	POSIA	TSIB	TSIA	TPIB	TPIB
PCA	1.00	.52	.25	.37	.12	.15	-.14	.02	.19	.26	.25	.18	-.05	-.03
PCB		1.00	.23	.42	.25	.13	-.19	-.18	.06	-.02	.09	-.01	-.08	-.07
POB			1.00	.33	-.13	-.03	.12	-.02	.48	.13	.34	.28	-.05	-.16
POA				1.00	-.02	.14	-.08	-.05	.00	.06	.25	.02	-.12	-.15
TB					1.00	.36	-.22	-.12	.03	-.15	-.22	-.08	-.28	-.22
TA						1.00	-.22	-.09	.06	-.16	-.15	-.17	-.03	-.05
PCSIA							1.00	.68*	.33	.37	.43	.51	.23	.02
PCSIB								1.00	.32	.36	.36	.51	.22	-.00
POSIB									1.00	.57	.58	.73*	.03	-.03
POSIA										1.00	.60*	.70*	.26	.27
TSIB											1.00	.72*	.02	.07
TSIA												1.00	.05	.12
TPIB													1.00	.66*
TPIA														1.00

*$p < .05$, using Sheffé procedure for multiple comparisons ($J = 14$) (i.e., $|Z| / \sqrt{1/(N-3)} \geq \sqrt{(J-1)1.96}$)
Note: See Appendix A.8 for Glossary of Variable Names.

A.10 POST HOC ANALYSIS OF 'Y' RESPONSES DURING POSTTESTS

As a post hoc exploratory analysis, the variables; the number of 'y' responses on POA (POAY) and the number of 'y' responses on POB (POBY), were subject to an analysis of variance. Table XVIII summarizes the results of the post hoc analysis of POAY and POBY. Figures 8 and 9 give the cell means and standard deviations and a plot of the cell means. There was a significant A effect and B effect for both POAY and POBY.

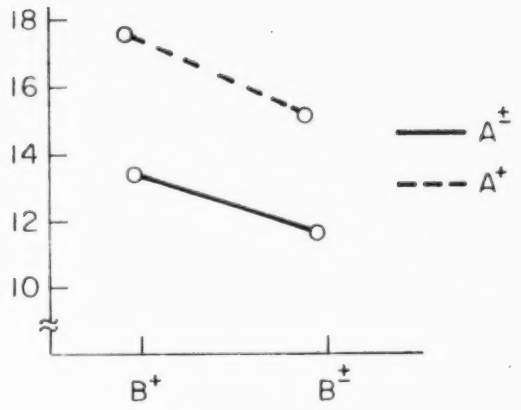
TABLE XVIII

POST HOC ANOVA OF THE NUMBER OF 'Y' RESPONSES ON POA AND POB

Variable	Effects	F	p <	Level Favored[a]		Transfer Effect
				+	±	
POAY	A x B	0.234	.630			
	A	23.076	.001**		Yes	
	B	7.390	.009**		Yes	Yes
POBY	A x B	3.647	.061			
	A	26.067	.001**		Yes	Yes
	B	31.062	.001**		Yes	

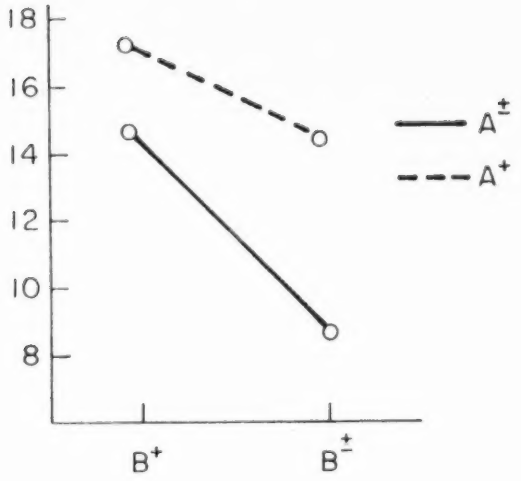
[a]Favored on number of 'y' responses means group favored made fewer 'y' responses.

*p < .05, **p < .01.



	A^+	$A^\pm$	
B^+	17.875	13.438	15.657
$B^\pm$	15.188	11.563	13.375
	16.532	12.500	

Figure 8. Cell means for POAY (A effect and B effect)



	A^+	$A^\pm$	
B^+	17.313	14.750	16.031
$B^\pm$	14.375	8.750	11.563
	15.844	11.375	

Figure 9. Cell means for POBY (A effect and B effect)